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Three Boundary Meshless Methods for Heat Conduction Analysis in Nonlinear FGMs with Kirchhoff and Laplace Transformation

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Abstract. This paper presents three boundary meshless methods for solving problems of steady-state and transient heat conduction in nonlinear functionally graded materials (FGMs). The three methods are, respectively, the method of fundamental solution (MFS), the boundary knot method (BKM), and the collocation Trefftz method (CTM) in conjunction with Kirchhoff transformation and various variable transformations. In the analysis, Laplace transform technique is employed to handle the time variable in transient heat conduction problem and the Stehfest numerical Laplace inversion is applied to retrieve the corresponding time-dependent solutions. The proposed MFS, BKM and CTM are mathematically simple, easy-to-programming, meshless, highly accurate and integration-free. Three numerical examples of steady state and transient heat conduction in nonlinear FGMs are considered, and the results are compared with those from meshless local boundary integral equation method (LBIEM) and analytical solutions to demonstrate the efficiency of the present schemes.

AMS subject classifications: 35k55, 41A30, 44A10, 65M70, 80M25

Key words: Method of fundamental solution, boundary knot method, collocation Trefftz method, Kirchhoff transformation, Laplace transformation, meshless method.

1 Introduction

Functionally graded materials (FGMs) are a class of composite materials whose microstructure varies from one material to another with a specified gradient. Due to

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their continuously graded properties, FGMs are superior to conventional composites and have featured in a wide range of engineering applications (e.g., thermal barrier materials [1], optical materials [2], electronic materials [3] and even biomaterials [4]).

Since FGMs are widely used for structures subjected to thermal loading, it is important to analyze their thermal behaviors. Analytical methods are usually restricted to simple physical domains and boundary conditions. Therefore, in the past decades, extensive studies have been carried out on developing numerical methods for analyzing thermal behaviors of FGMs, for example, the finite element method (FEM) [5], the boundary element method (BEM) [6, 7], the meshless local boundary integral equation method (LBIE) [8], the meshless local Petrov-Galerkin method (MLPG) [9,10] and the method of fundamental solution (MFS) [11, 12]. However, the conventional FEM is inefficient for handling materials whose physical property varies continuously; the BEM needs to treat the singular or hyper-singular integrals, which is mathematically complex and requires extensive computational resources. To avoid these drawbacks in the traditional FEM and BEM, various approaches [8–14] have been proposed, they are named as meshless method in the literatures. Among these meshless methods, the LBIE and the MLPG are classified as the category of weak-formulation, and the MFS is classified as the category of strong-formulation.

In this paper, we focus on meshless methods with strong-formulation. This is due to their inherent merits on easy-to-programming and integration-free. The MFS has to construct a fictitious boundary [15–17] outside the physical domain to avoid the singularities of fundamental solutions, however, selecting the appropriate fictitious boundary plays a vital role for the accuracy and reliability of the MFS solution. Herein the other two popular boundary collocation meshless methods are developed to avoid the singularities of fundamental solutions and the controversial fictitious boundary in the MFS. The first one is an old and powerful numerical scheme, collocation Trefftz method (CTM) [18], which chooses nonsingular T-complete functions as basis function. Therefore the boundary knots can be placed on the physical boundary. The second one is boundary knot method (BKM) proposed by Chen and Tanaka [19], which used the nonsingular radial basis function (RBF) general solution instead of the singular fundamental solution. Thus the boundary knots are also placed on the physical boundary.

On the other hand, the boundary meshless methods have been employed to deal with transient heat conduction problems through three different approaches: (1) time-dependent basis function method [20], one need to derive the corresponding basis function as a priori to satisfy the transient heat conduction equation and then solve it directly; (2) time stepping method [21], it transforms the transient heat conduction problem into time-independent inhomogeneous problem then introduces some additional particular techniques to solve this inhomogeneous problem; (3) Laplace transform technique [7], it uses the Laplace transformation of governing equation to eliminate the time derivative leading to a steady-state heat conduction equation in Laplace space, which can be solved by boundary meshless methods, and then employ numerical Laplace inversion scheme to invert the Laplace space solutions back into