Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **4**, No. 5, pp. 603-616

DOI: 10.4208/aamm.10-m11159
October 2012

The Collocation Method and the Splitting Extrapolation for the First Kind of Boundary Integral Equations on Polygonal Regions

Li Wang*

Civil Aviation Flight University of China, Guanghan 618307, China

Received 17 October 2011; Accepted (in revised version) 27 February 2012

Available online 30 July 2012

Abstract. In this paper, the collocation methods are used to solve the boundary integral equations of the first kind on the polygon. By means of Sidi's periodic transformation and domain decomposition, the errors are proved to possess the multi-parameter asymptotic expansion at the interior point with the powers $h_i^3 (i = 1, ..., d)$, which means that the approximations of higher accuracy and a posteriori estimation of the errors can be obtained by splitting extrapolations. Numerical experiments are carried out to show that the methods are very efficient.

AMS subject classifications: 65R10

Key words: Splitting extrapolation; boundary integral equation of the first kind on polygon; collocation method; posteriori estimation.

1 Introduction

By using the single layer potential theory, the plane Dirichlet problem

$$\begin{cases} \triangle u = 0, & (\Omega \quad \text{or} \quad \Omega^c), \\ u = h, & (\Gamma) \end{cases} \tag{1.1}$$

can be converted into a boundary integral equation of the first kind

$$-\frac{1}{\pi}\int_\Gamma g(P)\ln|P-Q|dS_Q = h(P), \quad \forall P \in \Gamma, \tag{1.2}$$

where Ω is a polygon, and Γ is its boundary. The Dirichlet problem on $\Omega^c = R^2/\Omega$ is called an exterior problem.

*Corresponding author.
Email: Joy81216060@163.com (L. Wang)

 ©2012 Global Science Press

We all know that the mathematical theory of the first kind of boundary integral equations is usually more difficult than the second kind due to lack of Fredholm alternative theorem. Although from the viewpoint of the calculation, the work of the discrete matrix generation and the accuracy of the approximation of the first kind of boundary integral equations are better than the second kind, but the mathematical theory of the first kind boundary integral equations is developed only by Sloan and Spence in [1] until 1988. They proved that if the capacity $C_\Gamma \neq 1$, then there was a unique solution in (1.2). Once $g(P)$ was solved, the solution of the interior problem (or exterior problem) can be expressed by

$$u(P) = -\frac{1}{\pi}\int_\Gamma \ln|P-Q|g(Q)dS_Q, \quad \forall P \in R^2\backslash\Gamma. \tag{1.3}$$

Sloan and Spence also used Galerkin method to solve the first kind boundary equations, and proved that using the Galerkin method, the accuracy of the interior-point approximations had superconvergence. However, the computational complexity of Galerkin method was too huge. Yan and other authors in [2] used the constant element collocation method to solve (1.2) and got the error estimate at the interior point with $O(h^{\beta+3/2})$, where $\beta = (1-\alpha)/\alpha$ and $\alpha\pi$ were the largest interior angle of Γ. This means that the accuracy reduces on concave regions. Thus, Yan in [3] recommended getting the high accuracy by mesh grading, which undoubtedly increased the difficulty of calculating. By using the mechanical quadrature method Lu Tao and Huang Jin in [4] proved the convergence of approximate solutions and the asymptotic expansions of the error, which can be used to accelerate the convergence by Richardson's extrapolation.

Splitting extrapolation method (SEM) based on a multivariate asymptotic expansion of the error is an effective parallel algorithm, which possesses high order of accuracy and high degree of parallelism (see [6]). By means of SEM, a large problem can be turned into many smaller discrete problems involving several grid parameters. If the errors of approximations of the problems have the multivariate asymptotic expansions, then after solving these small subproblems in parallel, the higher accuracy is computed by SEM.

In this paper, the collocation methods are used to solve the boundary integral equations of the first kind on the polygons. By means of Sidi's periodic transformation (see [5]) and domain decomposition, the errors are proved to possess the multi-parameter asymptotic expansion at the interior point with the powers $h_i^3 (i = 1, \cdots, d)$, which means that the approximations of higher accuracy and a posteriori estimation of the errors can be got by SEM.

In section 2, we will discuss the collocation method for the first kind of boundary integral equations on a circle. It will show that the error at the interior point have the asymptotic expansion. Based on section 2, further analysis for solving the first kind of boundary integral equations on a polygonal domain will be carried out. In section 3, using the results of the circle and the midpoint trapezoidal formula, the multi-parameter asymptotic expansion of the error at the interior point with the pow-