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Numerical Solutions of the System of Singular Integro-Differential Equations in Classical Hölder Spaces

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Abstract. New numerical methods based on collocation methods with the mechanical quadrature rules are proposed to solve some systems of singular integro-differential equations that are defined on arbitrary smooth closed contours of the complex plane. We carry out the convergence analysis in classical Hölder spaces. A numerical example is also presented.

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Key words: Collocation method, classical Hölder space, system of singular integro-differential equation, Fejér points.

1 Introduction

Singular integral equations (SIE) and singular integro-differential equations (SIDE) are used to model many application problems in electronics, aerodynamics, mechanics, thermo-elasticity, and queuing analysis, see for example, [1–7] and the literature cited therein. The general theory of SIE and SIDE has been studied in depth in the last decades, see for example, [8–10]. Analytic solutions of SIE and SIDE are rarely available. It is necessary to find approximate solutions for SIE and SIDE. There is rich literature about approximate solutions of SIE and SIDE using collocation methods with mechanical quadrature rules. Most of existing methods are designed for two cases: One is that the contour Γ is a unit circle; and the other case is that Γ is an interval on the real axis, see for example, [12–16]. In this paper, we study the case for which

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the contour of integration Γ can be an arbitrary smooth closed curve on the complex plane. It should be noted that conformal mapping from an arbitrary smooth closed contour with origin to the unit circle is not trivial and may make the problem more complicated due to the following reasons:

- The coefficients, kernel, etc. may become more complicated after the conformal mapping.
- The convergence analysis may be more difficult after the transform.
- Many existing numerical methods may not work for the transformed problem.

We note that the collocation method with the mechanical quadrature rules for one dimensional SIDE in generalized Hölder spaces has been investigated in [23–25]; and for SIE in classical Hölder spaces has been investigated in [20–22].

The convergence analysis of the collocation method with the mechanical quadrature rules for systems of SIDE in classical Hölder spaces has not been investigated when the equations are defined on an arbitrary smooth closed contour.

In this article we study the collocation method with mechanical quadrature rules for approximate solutions of systems of SIDE. The theoretical background of our proposed methods is based on Krikunov's integral representation and V. Zolotarevski's results for approximate solutions of SIE published in [9, 18–21].

The paper is organized as follows. In Section 2 we introduce some definitions and notations. We describe the integro-differential equations in Section 3. We present our numerical schemes of the collocation method with the mechanical quadrature rules and carry out the convergence analysis in Section 4. We show numerical example in Section 5.

2 Main definitions and notations

Let Γ be an arbitrary smooth closed contour bounding a simply connected region F^+ of the complex plane; and $t = 0 \in F^+$ and $F^- = \mathbb{C} \setminus \{F^+ \cup \Gamma\}$, where $\mathbb{C}$ is the complex plane. Let $z = \psi(w)$ be a function, mapping conformably the outside of unit circle $\Gamma_0 (= |w| = 1)$ on F^- such that

$$\psi(\infty) = \infty, \quad \psi^{'}(\infty) = c_0 > 0.^{\dagger} \tag{2.1}$$

By the Riemann mapping theorem the function $z = \psi(w)$ which satisfies the conditions (2.1) exists and it is unique [27]. There are a lot of such contours, see for example, Lyapunov contours [26] and others. We denote by Λ the set of the contours for which the relation (2.1) holds.

We denote by $[H_\beta(\Gamma)]_m$, $0 < \beta \leq 1$, the Banach space of m-dimensional vector-functions (v.f.), satisfying on Γ the Hölder condition with degree β. The norm is de-

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†We use $c_0, c_1, \cdots$, to represents constants.