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Binary Level Set Methods for Dynamic Reservoir Characterization by Operator Splitting Scheme

Changhui Yao*

Department of Mathematics, Zhengzhou University, No. 100 of Science Road, Zhengzhou 450001, Henan, China

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Abstract. In this paper, operator splitting scheme for dynamic reservoir characterization by binary level set method is employed. For this problem, the absolute permeability of the two-phase porous medium flow can be simulated by the constrained augmented Lagrangian optimization method with well data and seismic time-lapse data. By transforming the constrained optimization problem in an unconstrained one, the saddle point problem can be solved by Uzawas algorithms with operator splitting scheme, which is based on the essence of binary level set method. Both the simple and complicated numerical examples demonstrate that the given algorithms are stable and efficient and the absolute permeability can be satisfactorily recovered.

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Key words: Dynamic reservoir characterization, binary level set method, operator splitting scheme, the augmented lagrangian method.

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1 Introduction

We consider the conversation of mass for two-phase, incompressible, immiscible, horizontal flow in a porous medium with isotropic permeability:

$$\Phi(\mathbf{x})\frac{\partial S_o}{\partial t} - \nabla\cdot\Big(\frac{\kappa(\mathbf{x})\kappa_{ro}(S_o)}{\mu_o}\nabla p_o\Big) = f_o(\mathbf{x}), \tag{1.1a}$$

$$\Phi(\mathbf{x})\frac{\partial S_w}{\partial t} - \nabla\cdot\Big(\frac{\kappa(\mathbf{x})\kappa_{rw}(S_w)}{\mu_w}\nabla p_w\Big) = f_w(\mathbf{x}), \tag{1.1b}$$

where $(\mathbf{x},t)\in\Omega\times[0,T]$, $\Omega\subset R^2$ is a bounded reservoir, and the subscripts o and w refer to the phases, oil and water, respectively. Also S_i denotes the saturation, μ_i

*Corresponding author.
Email: chyao@lsec.cc.ac.cn (C. H. Yao)

the viscosity, p_i the pressure, f_i the external volumetric flow rate and κ_{ri} is the relative permeability, where i is the fluid phase. The porosity and the absolute permeability are given by $\Phi(\mathbf{x})$ and $\kappa(\mathbf{x})$, respectively, see, e.g., [7, 24].

Closuring the system is obtained through an assumption of a completely saturated medium

$$S_o + S_w = 1 \tag{1.2}$$

and an assumed known function P_c defining the capillary pressure,

$$p_o - p_w = P_c. \tag{1.3}$$

The quantities Φ, κ, κ_{ri} and P_c are all depended of the porous medium and are not accessible through direct measurements.

The considered problem is how to estimate an absolute permeability $\kappa(\mathbf{x})$, when Φ and κ_{ri} are assumed to be known, and P_c is set to zero. For recovering the permeability, we need utilize information from the wells together with seismic data. Unfortunately, we cannot get any direct information of permeability. However, through the Eqs. (1.1)-(1.3), we can use the indirect information to estimate the permeability on a coarse scale. Generally, such a problem can be called an inverse problem, or more specific referred to as a *history matching problem* [1, 27, 28]. In order to overcome the ill-posedness, regularization methods are always applied with different regularized terms.

The forward model (the solution of Eqs. (1.1)-(1.3) for a given function $\kappa(\mathbf{x})$) is solved by applying an in-house reservoir simulator. The simulator is using a standard block-centred grid with upstream weighting and Euler backwards in time discrete. Some numerical techniques can also cited in [9, 10].

In this work, we will use binary level set method [13, 17, 18] to recover the permeability. One of the essence of binary level set method is that we can constrain the solution to be a piecewise constant. And the geometries of the discontinuity of curves are allowed to be arbitrary, but with some constraint regularity achieved by a total variational regularization. For binary level set functions, the change of sign will show the discontinuity of the curves. From [14], we know that level set method can produce piecewise constant solution with a predefined number of constant levels. Practically, it can represent the sought solution with a few number of regions than the predefined number, which causes that one or more regions are empty [13]. Thus, we only need an upper bound of the number of regions in the piecewise constant solution.

This paper is based on the framework of [19]. The authors described the reservoir characterization using a binary level set approximation. They solved this problem by the augmented Lagrangian method, and used the gradient steepest descent method to get the next level set function values in their Uzawa algorithm. In [18], the authors also utilized data from wells and spatially distributed data with prior information about the sought solution to re-estimate the permeability. From the lights of work in [15, 16, 25], we can see that a better result can be obtained if operator splitting method is applied to inverse problem approximated with piecewise constant level set method without difficulties.