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Stability of Symmetric Solitary Wave Solutions of a Forced Korteweg-de Vries Equation and the Polynomial Chaos

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Abstract. In this paper, we consider the numerical stability of gravity-capillary waves generated by a localized pressure in water of finite depth based on the forced Korteweg-de Vries (FKdV) framework and the polynomial chaos. The stability studies are focused on the symmetric solitary wave for the subcritical flow with the Bond number greater than one third. When its steady symmetric solitary-wave-like solutions are randomly perturbed, the evolutions of some waves show stability in time regardless of the randomness while other waves produce unstable fluctuations. By representing the perturbation with a random variable, the governing FKdV equation is interpreted as a stochastic equation. The polynomial chaos expansion of the random solution has been used for the study of stability in two ways. First it allows us to identify the stable solution of the stochastic governing equation. Secondly it is used to construct upper and lower bounding surfaces for unstable solutions, which encompass the fluctuations of waves.

AMS subject classifications: 65C20, 65C30

Key words: Stability, solitary waves, polynomial chaos, forced Korteweg-de Vries equation.

1 Introduction

We investigate two-dimensional gravity-capillary waves in this paper. The analysis of properties of those waves such as the stability analysis has been one of main research areas in fluid mechanics, see [1–6] and the references therein. The Froude number F and the Bond number τ are important variables for the description of those waves. When F is close to unity and $\tau > 1/3$, the gravity-capillary waves can be modeled by the Korteweg-de Vries (KdV)-type equations. When the waves are generated by a

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localized pressure distribution or when the interfacial waves are considered in a spatial domain with bumps, forced KdV-type equations can be derived. Shen et al. in [7] derived a forced Korteweg-de Vries (FKdV) equation asymptotically. They found symmetric steady-state solitary-wave-like solutions and studied their stabilities. Choi et al. extended the study to derive a forced modified Korteweg-de Vries (FMKdV) equation in [8] and found symmetric steady-state solutions. The stability analysis of a damped KdV equation in a quarter plane was performed by Bona et al. in [9]. Larkin performed a mathematical analysis in [10] to prove the existence and uniqueness of strong and weak global solutions for the FMKdV equation in a bounded domain, and Pava and Natali [11] studied periodic traveling wave solutions for the critical KdV equation. Camassa and Wu [12, 13] performed the analysis of the stability for steady solitary-wave solutions and confirmed their analytical findings with accurate numerical computations. Grimshaw et al. [14] also performed the stability analysis on two-dimensional localized solitary waves from the steady forced KdV equation. Recently Chardard et al. derived solutions of the stationary forced KdV equation in [15] and Kim et al. computed the solutions of the forced modified KdV equation in [16].

Maleewong et al. derived the forced Korteweg-de Vries equation (2.1) in Section 2 in [17] and performed a stability analysis in [14]. In this study, we observe the evolutions of the solutions from this equation by perturbing its time-independent symmetric solitary-wave-like solutions. Our computation found four depression and one elevation time-independent solutions as Grimshaw et al. did in [14] and the simulation results show that the evolutions of the solution waves in time are dependent upon the magnitude of the perturbation. Thus, we regard the perturbation as a random variable in this study and we interpret the resultant governing equation as a stochastic differential equation. In this paper, we try to answer following two questions. First, is it possible to identify the stable solution, if it exists, among several time-independent solutions? Secondly, is it possible to estimate the magnitudes of the fluctuations of unstable solutions? Due to the random perturbation in the governing equation, the solution is a function of the deterministic and random variables. Cameron and Martin [18] proved that such a solution can be separated into deterministic and random variables by a Fourier series with respect to a certain polynomial chaos. Mikulevicius and Rozovskii considered problems with a random variable following a Brownian motion and performed theoretical analysis with respect to the polynomial chaos based on the Hermite polynomials in [19,20]. Ghanem and Spanos extended the study of the Gaussian stochastic process and the Hermite polynomial chaos to uncertainty problems in solid mechanics in [21, 22]. Xiu and Karniadakis [23] showed that optimal polynomial chaos is different when the distributions of the random variable is changed. For instance, the Hermite polynomial is optimal for the Gaussian random variable while the Laguerre polynomials is optimal for the Gamma random variable. Askey and Wilson classified the hypergeometric orthogonal polynomials for various types of random distributions and presented their properties in [24].

We derive a numerical algorithm for the forced Korteweg-de Vries equation. This research extends those of Grimshaw et al. [14] and Kim et al. [16], and the random