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Two-Phase Image Inpainting: Combine Edge-Fitting with PDE Inpainting

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Abstract. The digital image inpainting technology based on partial differential equations (PDEs) has become an intensive research topic over the last few years due to the mature theory and prolific numerical algorithms of PDEs. However, PDE based models are not effective when used to inpaint large missing areas of images, such as that produced by object removal. To overcome this problem, in this paper, a two-phase image inpainting method is proposed. First, some edges which cross the damaged regions are located and the missing parts of these edges are fitted by using the cubic spline interpolation. These fitted edges partition the damaged regions into some smaller damaged regions. Then these smaller regions may be inpainted by using classical PDE models. Experiment results show that the inpainting results by using the proposed method are better than those of BSCB model and TV model.

AMS subject classifications: 65M10, 78A48

Key words: Image inpainting, partial differential equations, edge fitting.

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1 Introduction

The task of filling in missing or damaged regions of an image is known as image inpainting which has many important applications in the area of digital image processing, visual analysis and film industry. The work has also been applied in repairing old photos, removing the scratches in old films, removing the redundant texts and objects in some scenes, magnifying images, image encoding, etc [1–3].

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The digital image inpainting technology based on partial differential equations (PDEs) has become an intensive research topic over the last few years [4–9] due to the mature theory and prolific numerical algorithms of PDEs. One basic PDE inpainting model is known as the BSCB model proposed by Bertalmio et al. [10,11]. The idea used in this model is propagating smoothly the information from the surrounding areas of the damaged region along the isophote directions to the damaged region. Chan and Shen [4,5] extended the classical TV denoising model of Rudin-Osher-Fatemi [12–14] and developed a total variation (TV) based inpainting model and the curvature driven diffusions (CDD) inpainting model from the point of view of variational principles and image prior models. In [15], Chan and Shen developed an Euler elastica (EE) inpainting algorithm based on connecting appropriate level lines by Euler elastica curves. This method turns out to be a generalization of the transportation mechanism of BSCB model and the CDD model.

However, as many researchers pointed out [16,17], these models are not effective when used to inpaint large missing areas of images, such as that produced by object removal. The reason is that the information of the outer borders of the missing areas can not be propagated into the inner pixels by current PDE models because they are far from the outer borders of missing areas [18]. To overcome this problem, in this paper, a two-phase image inpainting method is proposed. First, some edges which cross the damaged regions are located and the missing parts are fitted. These fitted edges partition the damaged regions into some smaller regions. Then these smaller regions can be inpainted by using above PDE models. Experiment results show that the inpainting results by using the proposed method are better than those of BSCB model and TV model.

The paper is organized as follows. First some classical PDE based inpainting models are reviewed. Then a two-phase inpainting method based on the combination of the cubic spline interpolation and PDE based inpainting is proposed. Finally numerical tests and some conclusions are given.

2 PDE-based image inpainting models

As depicted in Fig. 1, let Ω denote the entire domain of a damaged image, D the damaged region, where the image information is missing or damaged, $\Omega \setminus D$ the available part of the image, the inpainting boundary $\partial D \in \Omega \setminus D$.

The first PDE-based image inpainting model is proposed by Bertalmio et al., which can be described as below [10,11]:

$$\frac{\partial I}{\partial t} = \delta L \cdot \vec{N}, \tag{2.1}$$

where I is the image at time t; $\vec{N}$ represents the isophotes direction; δL is a measure of the change of certain information L being propagated along the isophotoes direction into the damaged region.

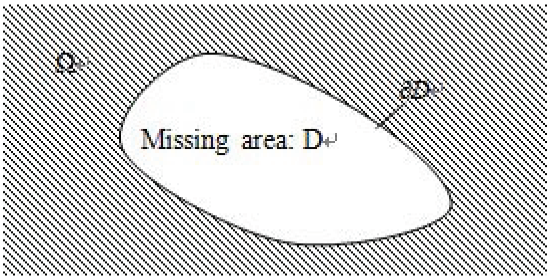

Figure 1: The domain of a damaged image and its damaged region.

Another common used PDE-based model is the total variation (TV) based model proposed by Chan and Shen [4, 5]. The idea is that an inpainted image I may get minimum for certain energy:

$$R(I) := \int_{\Omega} r(\|\nabla I\|)dxdy, \tag{2.2}$$

where the energy functional $r(s)$ should make $\int_{\Omega} r(\delta)dxdy$ finite in order to restore a broken edge where δ is a $1-D$ delta function.

Both these models can obtain good inpainting results when the damaged images are smooth and the damaged areas are small or they are just some cracks. But when the damaged region D is larger, the PDE models do not work.

3 Two-phase inpainting

According to the inpainting principle of PDE models given by [5], the model may be able to restore narrow broken smooth edges, but due to the large damaged region, the broken edge is wide and cannot be restored by PDE-based models. To avoid this problem, in this paper, a two-phase inpainting method is proposed. The main idea of the method is to fit these broken edges before inpainting the damaged regions and then these edges are used to partition the damaged regions into small regions. Finally these small regions are inpainted by using classical PDE models such as TV or BSCB model.

3.1 Edge fitting

As depicted in the Fig. 2, the edge e across the damaged region is broken into two parts: e_1 and e_2 which intersect the border ∂D of the damaged region D at p and q respectively. The first task is to find the two available parts e_1 and e_2 of the broken edge e and fit the broken part of e in the damaged region, as depicted in Fig. 3, where e_3 is fitted to connect e_1 and e_2.

There are many algorithms for finding the edges of an image. For example, Gradient operators such as Prewitt operator, Roberts operator can be used to obtain edges of images. Among these operators, the Canny edge detection method [19, 20] can get

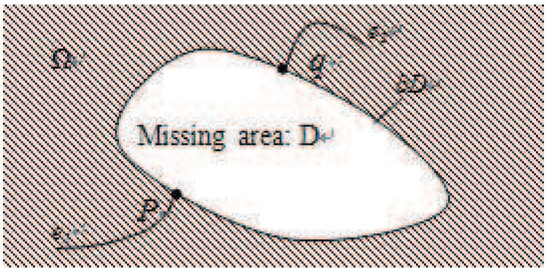

Figure 2: An edge cross the damaged region and is broken by the region.

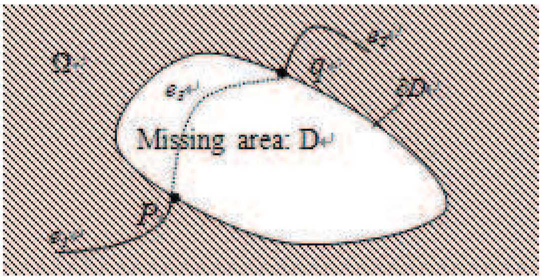

Figure 3: The broken edge in Fig. 3 is connected by the fitting method.

single-pixel edges of an image. In this paper, the Canny method is used to find the edges of the available region of the damaged image, for example, e_1 and e_2 depicted in Fig. 2.

In order to emphasize the idea of two-phase inpainting and describe the algorithm clearly, a single broken edge in the damage region is supposed in this paper. For the cases with a lot of broken edges, some criteria need to be established for matching two end points of a broken edge [21].

To fit the broken part e_3 of the edge e in the damaged region, it needs to locate the position of every pixel of e_3 in the damaged region first and then estimate the brightness of these pixels.

(1) Locating the position of the broken edge

The fitted broken part e_3 needs to connect e_1 and e_2 and make the fitted edge e as smooth as possible. The cubic spline interpolation [22,23] may be used to address this problem. In this paper, two one-variable cubic spline interpolations are used, one for x-axis coordinate and the other for y-axis coordinate.

A cubic spline S having $n+1$ knots $t_0, t_1, \cdots, t_n$ (real numbers satisfying $t_0 < t_1 < \cdots < t_n$) is a piecewise cubic polynomials such that:

a) On each interval $[t_{i-1}, t_i]$, S is a polynomial of degree ≤ 3.
b) S has a continuous 2nd derivative on $[t_o, t_n]$.

For a given table of data points as Table 1, the cubic spline interpolation is to construct a cubic spline which has the given value f_i in the knot t_i. To utilize the cubic spline interpolation to locate the broken edge in the damaged region, the key problem is how to construct the table of data points $l_i = (t_i, f_i)$, $0 \leq i \leq n$.

Table 1: Values of knots for cubic spline interpolation.

t	t_0	t_1	$\cdots$	t_n
f	f_0	f_1	$\cdots$	f_n

In the case of interaction with users, one can choose p as l_0, q as l_n and the other data points $l_1, l_2, \cdots, l_{n-1}$ are located in the damaged region, as depicted in the Fig. 4.

If the positions of data points need to be chosen automatically, the information of e_1 and e_2 may be used. As depicted in Fig. 5, choosing pixel points $p_u, p_{u-1}, \cdots, p_1, p$ ($u > 0$ is an integer) in the edge part e_1, a function f may be obtained by using the least square method to fit these points and the curve c_f which the function f represents will approximate edge part e_1. Choose some other points of c_f in the damaged region $p_{-1}, \cdots, p_{-v}$ ($v > 0$ is an integer) which as well as point p may be used as data points for the cubic spline interpolation. Similarly, some points $q_{-1}, \cdots, q_{-v}$ in the damaged region near to the edge part e_2 can be obtained which as well as q may also be used as data points for the cubic spline interpolation. That means, the data points for the cubic spline interpolation are $p, p_{-1}, \cdots, p_{-v}, q_{-v}, q_{-v+1}, \cdots, q_{-1}, q$.

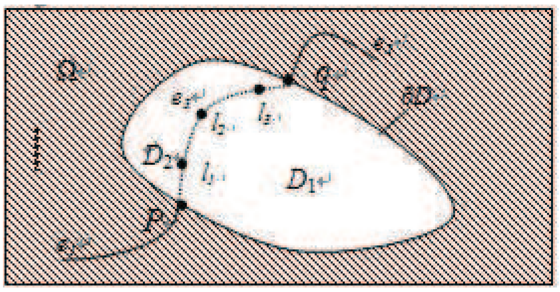

Figure 4: Choose interpolation knots by uses.

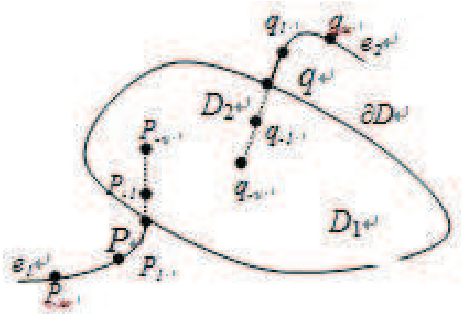

Figure 5: Fit and extend edge parts e_1, e_2 to the damaged region.

Thus the curve c_s represented by the cubic spline with data points p, $p_{-1}, \cdots, p_{-v}$, q_{-v}, $q_{-v+1}, \cdots, q_{-1}$, q may be used to approximate the broken part of the edge e.

The fitted curves described above are one-pixel width which may influence the effect of information propagation. In practice, the curves are broaden to 3-pixel or 5-pixel width to improve the inpainting effect.

(2) Estimation of intensities

As a part of an image, every point (pixel) of the curve c_s needs to be assigned an intensity. A simple fact is that, the intensities of the pixel points of an edge are similar and the closer two pixel points the more similar their intensities are. Based on these, the intensities of the edge point p and q at the border ∂D may be used to estimate the intensities of the other pixel points of the broken part e_3 in Fig. 4 or Fig. 5. Let the coordinates of p and q are (x_p, y_p) and (x_q, y_q) respectively and the intensities of p and q are I_p and I_q. Suppose l is a pixel point of e_3 whose coordinates are (x_l, y_l), the following simple interpolation is used to estimate the intensity I_l of the pixel point l:

$$\begin{cases} r_p = \sqrt{(x_l - x_p)^2 + (y_l - y_p)^2}, \\ r_q = \sqrt{(x_l - x_q)^2 + (y_l - y_q)^2}, \\ I_l = \dfrac{r_q}{r_p + r_q} I_p + \dfrac{r_p}{r_p + r_q} I_q, \end{cases} \tag{3.1}$$

where r_p and r_q are the distances from the pixel point l to p and q respectively.

3.2 PDE inpainting

After the broken edge is fitted, the damaged region D is partitioned into two small regions D_1 and D_2, as depicted in Figs. 4 and 5. If there is more than one broken edge in a damaged region or D_1, D_2 are still too large, the procedures described in Section 3.1 may be repeated to fit several broken edges which partitions D into several small regions. At this stage, PDE based inpainting models may be used to repair these small damaged regions. In this paper, BSCB model and TV model are used to inpaint these small damaged regions.

4 Experimental results

Five tests are designed to demonstrate the inpainting results of the new method proposed in this paper. In Test 1 and Test 2, the data points used for the cubic spline interpolation are chosen from the extension of the two available parts of one broken edge and the PDE model used for inpainting small damaged regions is TV model. In Test 3, the data points for the cubic spline interpolation are chosen by users manually, which may not locate in the extension of the two available parts of broken edges; and the PDE model used for inpainting small damaged regions is BSCB model. Test 4 and Test 5 give the examples of multiple broken edges in damaged areas.

In Test 1 the damaged image is a circle, as shown in Fig. 6.1. At first, the broken edge is found (Fig. 6.2), then the broken part is fitted and assigned intensities (Fig. 6.3), finally the two small damaged regions are inpainted by using the TV model (Fig. 6.4). Compared with the inpainting results by using the TV model (Fig. 6.5) and the BSCB model (Fig. 6.6), the result by using our method is perfect.

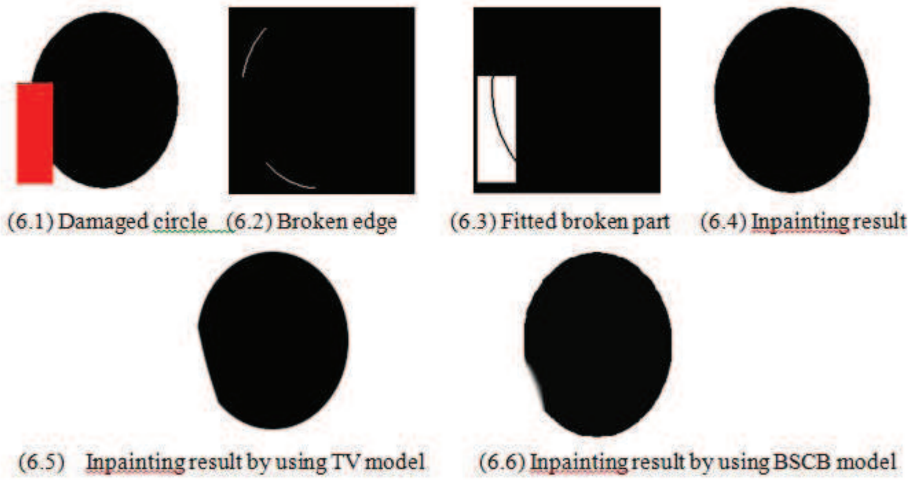

Figure 6: The process of repairing a damaged circle by using our method.

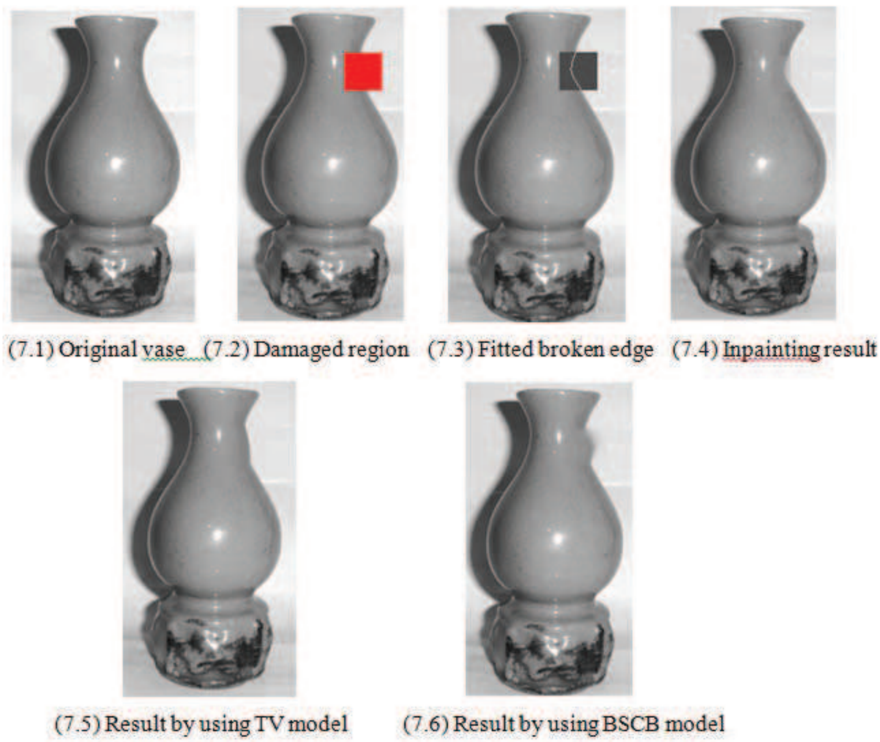

Figure 7: Process of repairing a vase by our method and the result comparison of three methods.

In Test 2, the original image and its damaged region are shown in Figs. 7.1 and 7.2. The positions of data points used for the cubic spline interpolation are obtained from the extension of the two available parts of the broken edge, as depicted in Fig. 7.3. The inpainting result by using the TV model to repair the small regions is shown in

Table 2: The data points (x, y) using for the cubic spline interpolation.

x	28	27	28	29	30
y	82	92	99	106	113

Table 3: The data points using for the cubic spline interpolation $x = x(t)$.

t	1	2	3	4	5
x	28	27	28	29	30

Table 4: The data points using for the cubic spline interpolation $y = y(t)$.

t	1	2	3	4	5
y	82	92	99	106	113

Fig. 7.4, which is better than the inpainting results by using the classical TV model (Fig. 7.5) and the BSCB model (Fig. 7.6).

Test 3 demonstrates the inpainting results by choosing data points manually to fit broken edges. The damaged region (Fig. 8.1) includes an edge which distinguishes the mountain from the sky. The data points for the cubic spline interpolation are shown in Table 2, which are the coordinates of some pixels located in the damaged region. The broken edge depicted in Fig. 8.2 is fitted as a curve with parameter t by using Table 3 and Table 4 which are constructed from Table 2. The result by using the BSCB model to inpaint the two small regions is shown in Fig. 8.3.

Note that the inpainting effect is not acceptable. Looking back the original damaged region in Fig. 8.1, one may find that there are still some implicit broken edges, for example, the edge distinguishing the cloud and the sky and the edge distinguishing the dark mountain from the light mountain. So the other two fitted edges (Fig. 8.4) are used to get a better inpainting result (Fig. 8.5), which is also better than the inpainting results by using the classical BSCB model.

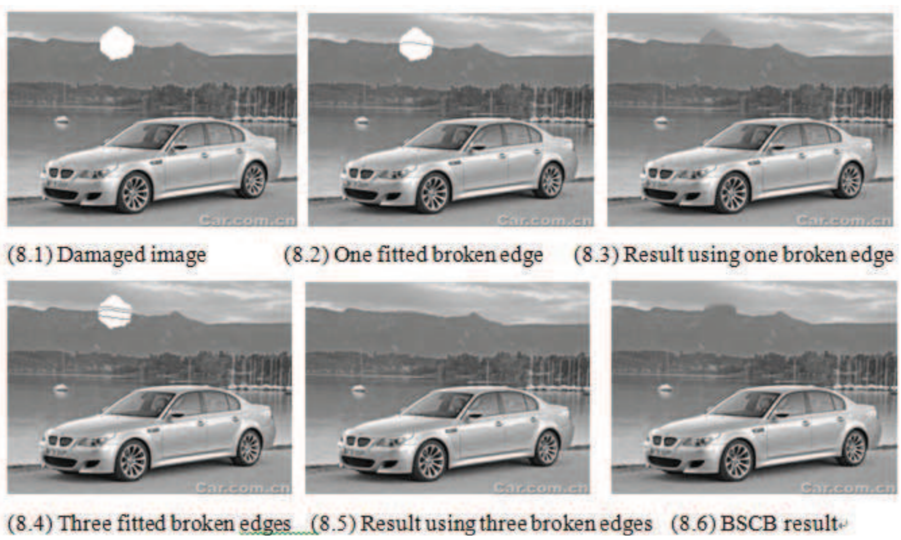

Figure 8: Repairing effects of different numbers of broken edges and comparison with BSCB result.

Figure 9: Process of repairing Lena image by our method and comparison with TV method.

Figure 10: Process of repairing the vase image by our method and comparison with TV method.

Tests 4 and 5 show more inpainting results of proposed methods compared with TV methods. In Test 4, the original Lena image and its damaged region are shown in Figs. 9.1 and 9.2. The inpainting result by using TV method is shown in Fig. 9.3. There are two broken edges which are shown in Fig. 9.4. The fitted lines are shown in Fig. 9.5 which partition the damaged area into three small regions. The small regions are inpainted by using TV model and the final inpainting result is shown in Fig. 9.6. One can find that the broken pole is repaired very well in the proposed method.

In Test 5, there is a brand mark in the original image (Fig. 10.1) and we wish to remove it (red part in Fig. 10.2). The inpainting result by using TV method is shown in Fig. 10.3. In the proposed method, the broken edges are located (Fig. 10.4) and then some curves are fitted to connect them, as depicted in Fig. 10.5. Finally, The small regions partitioned by the fitted curves are inpainted by using TV model and the inpainting result is shown in Fig. 10.6, which is better than the inpainting result by using the classical TV model (Fig. 10.3).

5 Conclusions

In this paper, a two-phase image inpainting method combining the cubic spline interpolation with PDE based inpainting methods is proposed to overcome the shortcoming that PDE based inpainting method cannot repair large damaged regions. In this novel method, some broken edges which cross the damaged regions are fitted by using the cubic spline interpolation. These fitted edges partition the damaged regions into some smaller damaged regions which can be repaired by using classical PDE based inpainting methods. Some experiments show the inpainting results by using the proposed method are better than those of BSCB model and TV model.

Note that the selection of data points for the cubic spline interpolation may be difficult. When the broken edges are simple, for example, they are not interacted, the data points for the cublic spline interpolation may be selected from the extension of the two available parts of the broken edges. If the broken edges are complex, the data points for the cublic spline interpolation may be selected by users in the damaged regions.

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