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Meshless Collocation Method for Inverse Source Identification Problems

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Abstract. A novel meshless scheme is proposed for inverse source identification problems of Helmholtz-type equations. It is formulated by the non-singular general solutions of the Helmholtz-type equations augmented with radial basis functions. Under this meshless scheme, we can determine smooth source terms from partially accessible boundary measurements with accurate results. Numerical examples are presented to verify validity and accuracy of the present scheme. It is demonstrated that the present scheme is simple, accurate, stable and computationally efficient for inverse smooth source identification problems.

AMS subject classifications: 65M10, 78A48

Key words: Meshless method, Helmholtz equation, source function.

1 Introduction

Inverse source problems are very important in many branches of science and engineering. They have attracted great attentions from many researchers because of their applications to many practical examples. Many theoretical and numerical studies have been executed over recent years. For an overview of this branch, we refer readers to references [8, 14, 21, 23, 25] and references therein.

Most of the previous numerical works are focused on the boundary element method [10]. Ohnaka and Uosaki [27] developed a boundary element approach for identifying the external force applied to a number of points on the domain of a distributed parameter system. By using DRM, Kagawa et al. [19] considered the identification of unknown electric charge distribution. Trlep et al. [31] presented the use of the dual

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reciprocity boundary element method (DRM) for the solution of the inverse problem described with Poisson's equation. The boundary element method is also used to compute the unknown surface velocity distribution on a complex acoustic source from measured acoustic field data [30]. The use of the inverse boundary element method (BEM) to identify aeroacoustic noise sources is explored in [24].

One of the boundary-type meshless method, the method of fundamental solutions [9, 15, 22], is shown to be superior to the boundary element method. It is becoming very popular for solving both direct and inverse problems [1, 6, 29]. By using the method of fundamental solutions, Jin and Marin considered the inverse source potential problem $\nabla^2 u(\mathbf{x}) = f(\mathbf{x})$ associated with the steady-state heat conduction with the extra assumption that $\Delta f = 0$ [18]. Their main idea is to, by the fact that f is harmonic, transform the original problem into a fourth-order partial differential equation. The resulting partial differential equation can be solved using numerical methods. Recently, Wang and his coworkers [35] considered a generalized case in a sense that the unknown source f need not be harmonic.

Note that most previous literatures are focused on the indirect approaches, i.e., the dual reciprocity method is used during the solution process. In this paper, a direct meshless collocation formulation is proposed for inverse smooth source problems associated with Helmholtz-type equations. The fictitious boundary used in [35] is eliminated in the current proposed method. Our problem is, first to give the meshless collocation formulation, then to study the effect of related parameters and stability issue from partially accessary boundary data.

The outline of this paper is as follows. In Section 2, we briefly state the inverse source problems. Section 3 established the meshless collocation formulation in terms of nonsingular general solutions of the Helmholtz-type equations augmented with radial basis functions. To deal with the ill-conditioned interpolation matrix system, we employ the Tikhonov regularization (TR) method under parameter choice of generalized cross validation. Numerical examples are discussed for both circular and irregular domains in Section 4. Section 5 concludes this paper with some additional remarks.

2 Problem description

Most acoustic sources encountered in real applications always have definite characters whcih can be modeled to the following Helmholtz governing equation

$$\nabla^2 u(\mathbf{x}) + k^2 u(\mathbf{x}) = f(\mathbf{x}), \quad \mathbf{x} \in \Omega, \tag{2.1}$$

under boundary conditions $u(\mathbf{x}) = \bar{u}(\mathbf{x})$ on Dirichlet boundary Γ_D and $q(\mathbf{x}) = \bar{q}(\mathbf{x})$ on Neumann boundary Γ_N, where $\bar{u}(\mathbf{x})$ and $\bar{q}(\mathbf{x})$ are pre-defined functions. $u = u(\mathbf{x})$ is the unknown field function, Ω is the physical domain with physical boundary $\partial\Omega$ $(= \Gamma_D \cup \Gamma_N)$, k and n are wavenumber and unit normal vector, respectively. In cases that the boundary is only partially accessible, the known functions $\bar{u}(\mathbf{x})$ and $\bar{q}(\mathbf{x})$ are given boundary conditions on $\Gamma \subset \partial\Omega$ simultaneously to ensure that the boundary value problems are solvable. We aim to determine the unknown source function $f = f(\mathbf{x})$ under the circumstances.