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Differential Quadrature Analysis of Moving Load Problems

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Abstract. The differential quadrature method (DQM) has been successfully used in a variety of fields. Similar to the conventional point discrete methods such as the collocation method and finite difference method, however, the DQM has some difficulty in dealing with singular functions like the Dirac-delta function. In this paper, two modifications are introduced to overcome the difficulty encountered in solving differential equations with Dirac-delta functions by using the DQM. The moving point load is work-equivalent to loads applied at all grid points and the governing equation is numerically integrated before it is discretized in terms of the differential quadrature. With these modifications, static behavior and forced vibration of beams under a stationary or a moving point load are successfully analyzed by directly using the DQM. It is demonstrated that the modified DQM can yield very accurate solutions. The compactness and computational efficiency of the DQM are retained in solving the partial differential equations with a time dependent Dirac-delta function.

AMS subject classifications: 74S30, 65M99

Key words: Differential quadrature method, Dirac-delta function, moving point load, dynamic response, work-equivalent load.

1 Introduction

The differential quadrature method (DQM), originated by Bellman and his associates in the early 1970s [1,2], is a numerical technique for the solution of initial as well as boundary value problems. Since Bert and his coworkers first used the DQM to solve problems in structural mechanics in 1988 [3], the method has been well developed and applied successfully to a variety of initial and boundary value problems. Several efficient DQ

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based time integration schemes are proposed by Fung [4,5], Shu et al. [6] and Chen and Tanaka [7]. The introduce of the special matrix product by Chen et al. [8] further simplifies the presentation of DQ equations and makes the DQM more efficient for solving non-linear problems. The DQM has been used in the fields of mathematics, physics and engineering. It is demonstrated that the DQM has the advantages of compactness and computational efficiency and can generate highly accurate numerical solutions. A vast body of literature exists. Relative research work in the area of structural mechanics and in general engineering has been well documented by Bert and Malik [9], Shu [10], Zong and Zhang [11] and Wang [12].

It is well known that the properties of the Dirac-delta function are not in the form of derivatives but in the form of integrals. Similar to the conventional point discrete methods such as the collocation method and finite difference method, the DQM has some difficulty in dealing with singular functions such as the Dirac-delta function. Numerical solutions may become oscillatory around the Dirac-delta function. To overcome this difficulty, the Dirac-delta function should be regularized to achieve a smoother representation and to stabilize the unwanted oscillatory behavior of solutions near the Dirac-delta function. A level-set finite difference approach for regularization of the Dirac-delta function is presented in [13].

From the literature review, it is rare to use the DQM directly for solutions of partial differential equation involving the Dirac-delta functions such as the problems of beams under a stationary or moving point load. For static analysis, the wavelet-based DQM [14] can yield reasonably accurate solutions by distributing the point load over a small region. Han et al. [15] also investigate the effect of different treatments of the concentrated load on the accuracy of DQ solutions. To obtain the equivalent load, linear, parabolic and cubic distributions are tried. They conclude that the solution accuracy is the highest if the point load is linearly distributed over a small region. The linear distribution is actually equivalent to the method used in [14] to deal with the concentrated load if the load is applied on a grid point. It is seen that the method to deal with the point load in [14,15] seems only physically sound but not mathematically clear. Over how small area should the load be uniformly or linearly distributed? Although the differential quadrature element method (DQEM) [16] can efficiently solve the static problem with a stationary point load, however, the DQEM is inconvenient for solving the moving loads problems which involve time dependent Dirac-delta functions, especially when several moving loads are involved.

To circumvent the difficulty encountered in dealing with the time-dependent Dirac-delta functions by using the DQM, the mixed Ritz-DQ method [17] and the mixed finite element-differential quadrature [18] are proposed. In these hybrid methods, Ritz method or finite element method is used to handle the time-dependent Dirac-delta functions and the DQM is merely used for the step by step time integration. Thus, directly using the DQM to solve the moving point load problem has not been reported thus far.

The objective of this paper is to propose a way to overcome the difficulty encountered in handling the time-dependent Dirac-delta function by using the DQM. Two modifica-