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Osher Flux with Entropy Fix for Two-Dimensional Euler Equations

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Abstract. We compare in this paper the properties of Osher flux with O-variant and P-variant (Osher-O flux and Osher-P flux) in finite volume methods for the two-dimensional Euler equations and propose an entropy fix technique to improve their robustness. We consider both first-order and second-order reconstructions. For inviscid hypersonic flow past a circular cylinder, we observe different problems for different schemes: A first-order Osher-O scheme on quadrangular grids yields a carbuncle shock, while a first-order Osher-P scheme results in a dislocation shock for high Mach number cases. In addition, a second-order Osher scheme can also yield a carbuncle shock or be unstable. To improve the robustness of these schemes we propose an entropy fix technique, and then present numerical results to show the effectiveness of the proposed method. In addition, the influence of grid aspects ratio, relative shock position to the grid and Mach number on shock stability are tested. Viscous heating problem and double Mach reflection problem are simulated to test the influence of the entropy fix on contact resolution and boundary layer resolution.

AMS subject classifications: 65M06, 65M70

Key words: Osher flux, entropy fix, Euler equation, finite volume method, ”carbuncle” shock.

1 Introduction

Upwind schemes based on the characteristic theory have been widely used in solving the Euler equations for their capability of capturing discontinuities. Upwind schemes are usually classified into flux-vector and flux-difference splitting schemes. Flux-difference

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splitting schemes such as Osher scheme [1,2] and Roe scheme [3] have gained popularity as they are robust and can capture steady discontinuities. Since the Osher scheme can capture steady discrete shock sharply [4–7], it has been widely used in fluid dynamics simulations [8–11]. There have been many numerical evidences showing the superiority of the Osher flux. For instance, Qiu et al. has done lots of work in [12–14] to compare the Osher fluxes with others recently, including Lax-Friedrich (LF), Godunov, Harten-Lax-van Leer (HLL), HLLC which is a modification of the HLL. They found that among these considered fluxes the Osher flux is the most robust one and has the best resolution near discontinuities. In addition, the Osher flux is also compared with the LF flux and the Vijayasundaram flux by Felcman and Havle in [15], which also shows that the numerical results obtained by the Osher scheme is the most accurate one.

For a supersonic blunt body problem, it was shown that the Roe's splitting produces a "carbuncle" schock (see [16, 17]). To solve this problem many efforts have been made. For instance, Lin proposed a cure by adding dissipation in linear waves to Roe's Riemann solver [18]. Sanders developed H-corrections based on the difference of the characteristic speeds on five interfaces [19]. Pandolfi and Ambrosio added appropriate dissipation according to the maximum difference between the characteristic speeds across the four interfaces [20]. Ren proposed a rotated Roe scheme based on the rotated Roe's approximate Riemann solver [21]. For Euler equations, the Osher flux takes a path integral of flux derivative in phase space. The integral subpaths are constructed based on the eigenvalues in decreasing order by Osher [4] and in increasing order by Spekreijse [22], which are referred to as the Osher-O flux and the Osher-P flux, respectively. There are already some investigations about the properties of these two different fluxes. For instance, Jacobs found that it is difficult to apply the Osher scheme for strong shock computations and proposed a 3-stage approximate Riemann solver at the cost of computations [23]. Amaladas and Kamath compared six kinds of upwind schemes based on steger-Warming, van Leer, Roe, Osher-P, AUSM and HUS, respectively [24]. It is indicated that the Osher schemes with O-variant and P-variant have different performance in inviscid hypersonic flow past a circular cylinder under structured mesh and the Osher scheme with O-variant can not result in a converged solution. However, detail comparisons of the two fluxes and the treatment of instability have not been investigated yet, which is the motivation of this paper.

The purpose of this paper is to compare the performance of the Osher-O and Osher-P fluxes for the two-dimensional (2D) Euler equation and to improve their shock robustness. Firstly, the Osher flux is combined with first-order and second-order reconstructions, respectively, to construct finite volume methods. Secondly, for inviscid hypersonic flow past a circular cylinder, the Osher schemes are investigated from the aspects of different grids and different Mach numbers. We find that the first-order Osher scheme will produce a "carbuncle" shock or "dislocation" shock and the second-order Osher scheme becomes unstable at the head of the cylinder. In order to improve the robustness of the above numerical methods, we propose an entropy fix technique, which adds dissipation based on the maximum difference of the characteristic speeds across the four interfaces.