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The LMAPS Using Polynomial Basis Functions for Near-Singular Problems

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Abstract. In this paper, we combine the Local Method of Approximate Particular Solutions (LMAPS) with polynomial basis functions to solve near-singular problems. Due to the unique feature of the local approach, the LMAPS is capable of capturing the rapid variation of the solution. Polynomial basis functions can become very unstable when the order of the polynomial becomes large. However, since the LMAPS is a local method, the order does not need to be very high; an order of 5 can achieve sufficient accuracy. In order to show the effectiveness of the LMAPS using polynomial basis functions for solving near singular problems, we compare the results to the LMAPS using radial basis functions (RBFs). The advantage of using polynomials as a basis rather than RBFs is that finding an appropriate shape parameter is not necessary.

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Key words: Method of particular solutions, near-singular problems, polynomial basis functions, localized method of approximate particular solutions.

1 Introduction

The method of particular solutions (MPS) [5, 6, 23] has become a widely used meshless method for the numerical solutions of partial differential equations. Knowing that the particular solution is not unique, a variety of techniques have been developed to find a particular solution [1,6,7,13,18,21] for differential operators and basis functions. The development of the MAPS has been primarily restricted to the use of radial basis functions (RBFs). While RBFs have been used with wide success [11], challenges remain including the determination of an appropriate shape parameter and deriving the closed-form

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particular solutions. To alleviate these difficulties, Chebyshev polynomial functions have been applied which has shown to be highly accurate [15, 19, 21]. However, the closed-form particular solution is limited to only specific differential operators. Also the forcing term must be smoothly extendable to the exterior of the domain. Dangal and Chen [8] recently derived the particular solution using the polynomial basis function under a general linear differential operator. By coupling this with the MAPS, a wide class of partial differential equations has been solved. A distinct advantage of this approach is that the collocation points can have an arbitrary distribution inside the computational domain without requiring the use of fictitious collocation points outside the domain.

In this paper, we are interested in solving near-singular problems. To our knowledge, none of the global methods, such as the Kansa method, the method of fundamental solutions (MFS) [9], singular boundary method [17] and the MPS, can be used successfully for solving near-singular problems. The globally defined RBFs have been shown to produce unacceptable results in [4]. The traditional meshed based methods such as finite element method [2, 3, 12] and finite difference method [25] are tedious in producing the meshes in the solution process. Additionally, polynomial basis functions become notoriously unstable as the order of the polynomial grows large. In particular, the method of approximate particular solutions (MAPS) in polynomial basis functions needs a high polynomial order. For near-singular problems, the MAPS in polynomial basis functions becomes more difficult near the singular point. However, a localized scheme can be applied to the MAPS, known as the local method of approximate particular solutions (LMAPS), which is capable of capturing the rapid variation of the solution. Hence, the LMAPS is attractive for solving near-singular problems as shown in [4]. When using the LMAPS with RBFs, an optimal shape parameter [10] must be chosen in order to yield acceptable accuracy [10]. This process is not trivial and is an ongoing research problem. In this paper, we will couple the LMAPS with polynomial basis functions to avoid the issue of choosing an appropriate shape parameter. Since it is a local method, it is not necessary for the order of the polynomial to be very high. This method will then be applied to various near-singular problems in two- and three-dimensions on regular and irregular domains.

The paper is organized as follows. In Section 2, we derive the closed-form particular solution for the general differential equation using polynomial basis functions. In Section 3, we give a brief review of the LMAPS in the context of polynomial basis functions. In Section 4, to demonstrate the effectiveness of the LMAPS using polynomial basis functions for solving near-singular problems, we present the results of five numerical examples and compare the results with the LMAPS using RBFs. Finally, some conclusions and ideas for future work are outlined in Section 5.

2 Particular solution of polynomial basis functions

In this section, we consider polynomial basis functions and find the particular solutions of the basis functions for general partial differential operators in both two- and three-