Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **10**, No. 5, pp. 1126-1157

DOI: 10.4208/aamm.OA-2017-0272
October 2018

The Plane Wave Methods Combined with Local Spectral Finite Elements for the Wave Propagation in Anisotropic Media

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Received 21 October 2017; Accepted (in revised version) 12 February 2018

Abstract. The plane wave least-squares method combined with local spectral finite elements has been used effectively to solve time-harmonic acoustic and electromagnetic wave propagation with complex wavenumbers. We develop the plane wave least-squares method and the ultra weak variational formulation for the nonhomogeneous case of the electromagnetic wave propagation in anisotropic media. We derive error estimates of the approximation solutions generated by these methods in one special case of TE mode scattering. Numerical results indicate that the resulting approximate solutions generated by these two methods possess high accuracy and verify the validity of the theoretical results.

AMS subject classifications: 65N30, 65N55

Key words: Electromagnetic wave, plane wave least-squares, ultra weak variational formulation, anisotropic media, plane-wave basis, error estimates.

1 Introduction

The plane-wave method, which falls into the class of Trefftz methods [24], was first introduced to solve Helmholtz equations and was then extended to solve Maxwell's equations and time-harmonic elastic wave propagation. Examples of this approach include the partition of unity-type method [6,7], the variational theory of complex rays (VTCR) [21,

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22], the ultra weak variational formulation (UWVF) [3, 4, 8, 15–17, 26], the plane-wave discontinuous Galerkin (PWDG) method [9, 11], the plane-wave least-squares (PWLS) method [12, 13, 20, 25, 27] and the plane-wave least-squares combined with local spectral finite element (PWLS-LSFE) method [14]. The plane wave methods have an important advantage over the other methods for discretization of the Helmholtz equation and time-harmonic Maxwell equations: the resulting approximate solutions have higher accuracies, owing mainly to the choice of the basis functions satisfying the governing differential equation without boundary conditions.

Since plane wave basis functions on each element are solutions of the *homogeneous* Helmholtz equation or time-harmonic Maxwell equations without boundary condition, the plane wave methods can not be directly applied to discretizations of the *nonhomogeneous* Helmholtz equation and time-harmonic Maxwell equations. Fortunately, the plane wave method combined with local spectral elements (PWLS-LSFE) for the discretization of such nonhomogeneous equations was firstly proposed in [14]. This method contains two steps: a series of nonhomogeneous local problems on auxiliary smooth subdomains are solved by the spectral element method and then the plane wave method are employed to discretize the resulting (locally homogeneous) residue problem on the global solution domain. The numerical results show that the resulting approximate solution is more accurate than that generated by the PWLS method [13].

Recently, the UWVF method was extended to solve homogeneous Maxwell's equations in isotropic media in [15] and in anisotropic media in [16], respectively. The studies [15] were devoted to computing the electric and magnetic fields in a non-absorbing medium or within the PML. The studies [16] were devoted to approximating the Robin-type trace of the the electric and magnetic fields in an anisotropic medium or within the PML and focus on the numerical test and convergence analysis in TM mode scattering, which can result in a Helmholtz equation in two dimensions with an anisotropic coefficient.

In this paper we are mainly interested in extending the PWLS-LSFE in an isotropic medium to the nonhomogeneous case in an anisotropic medium for three-dimensional system of Maxwell's equations. Moreover, we derive error estimates of the approximate solutions generated by the PWLS-LSFE in one special case. Motivated by the UWVF mthod developed in [16], we develop a new variant of the UWVF, which can directly obtain the approximations to the electromagnetic filed by choosing different trial functions and test functions. For convenience, we call the resulting variational formulation as UWVF-LSFE. The same procedure can also be generalized to the PWDG method developed in [10, 11].

Numerical experiments show that the approximate solutions generated by the PWLS-LSFE and the UWVF-LSFE method for three-dimensional Maxwell's equations possess almost the same and satisfactory accuracy and verify the validity of the theoretical results in the TM mode case. To efficiently solve the algebraic system generated by the PWLS-LSFE method, the non-overlapping domain decomposition preconditioner introduced in [13] is applied for the resulting stiffness matrix.