Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **5**, No. 2, pp. 146-162

DOI: 10.4208/aamm.12-m1224
April 2013

Numerical Approximation of Hopf Bifurcation for Tumor-Immune System Competition Model with Two Delays

Jing-Jun Zhao*, Jing-Yu Xiao and Yang Xu

Department of Mathematics, Harbin Institute of Technology, Harbin 150001, Heilongjiang, China

Received 20 March 2012; Accepted (in revised version) 20 July 2012

Available online 28 January 2013

Abstract. This paper is concerned with the Hopf bifurcation analysis of tumor-immune system competition model with two delays. First, we discuss the stability of state points with different kinds of delays. Then, a sufficient condition to the existence of the Hopf bifurcation is derived with parameters at different points. Furthermore, under this condition, the stability and direction of bifurcation are determined by applying the normal form method and the center manifold theory. Finally, a kind of Runge-Kutta methods is given out to simulate the periodic solutions numerically. At last, some numerical experiments are given to match well with the main conclusion of this paper.

AMS subject classifications: 34K18, 37G10, 37G15, 37N25

Key words: Hopf bifurcation, delay, tumor-immune, dynamical system, periodic solution.

1 Introduction

At present, cancer is still a leading cause of death in the world, even if that is still not known about its mechanisms of establishment and destruction. In many cases, surgery is not represent a cure. Many patients can not find the tumor in time, then later degeneration can occur. The theoretical study of tumor-immune dynamics has a long history [1]. A detailed description of virus, antivirus, and body dynamics can be found in [2–4].

Immune system plays a key role in the initial stage when tumor occurs. The immune system responses consist of two different interacting responses: the cellular response and the humoral response. The cellular response is carried by T lymphocytes. The humoral response is related to the other class of cells, called B lymphocytes. A dynamics of the

*Corresponding author.
Email: hit_zjj@hit.edu.cn (J.-J. Zhao)

antitumor immune response in vivo is complicated and not well understood. A number of mathematical models of the interactions between the immune system and a growing tumor have been developed [5, 6]et al. The kinetics of cell mediated cytotoxicity in vitro have also been described by mathematical models [7, 8]et al.

The mathematical model with which we are dealing, was proposed in papers by Galach [9] and Yafia [10]. In the former paper, the author developed a simple model of tumor immune system competition without delay, whose idea was inspired from [11]. It is shown that this model had a nonnegative periodic solution when the parameters changed. In the latter paper, the author published a series of papers to analyze the Hopf bifurcation problem which predict the occurrence of a limit cycle bifurcation.

The aim of this paper is to show that the tumor-immune dynamics with two delays has a Hopf bifurcation as the time delays changed. The existence of critical values of the delays are investigated, in which stability of the nontrivial steady states changed. Main result of this paper is given in Section 3. Based on the Hopf bifurcation theorem, we show the occurrence of Hopf bifurcation when the delay crosses some critical value. In Section 4, we determine the direction and stability of the branch of periodic solutions bifurcating from the nontrivial steady state by using the theory presented in Hassard et al. [12]. In Section 5, we construct Runge-Kutta methods with the interpolation procedure for the system with two delays. Finally in Section 6, we give some numerical examples to show that the Hopf bifurcation can occur at some critical values. Moreover, numerical comparisons are made between our Runge-Kutta methods and dde23 function in matlab.

2 Mathematical model

The Kuznetsov and Taylor's model describes the response of effector (ECs) to the growth of tumor cells (TCs). This model differs from others because it takes into account the penetration of TCs by ECs, which simultaneously causes the inactivation of ECs. It is assumed that there exist interactions between ECs and TCs in vitro, which can be described by the kinetic scheme shown in Fig. 1, where E, T, C, E^*, and T^* are the local concentrations of ECs, TCs, EC-TC complexes, inactivated ECs, and "lethally hit" TCs, respectively. Here, k_1 and k_{-1} denote the rates of bindings of ECs to TCs and the detachment of ECs

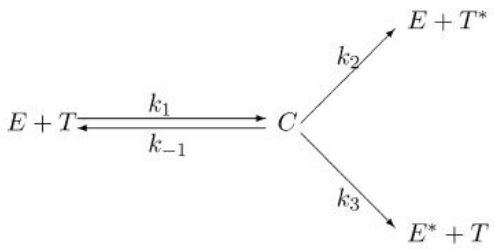

Figure 1: Kinetic scheme describing interactions between ECs and TCs.