Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **5**, No. 2, pp. 222-234

DOI: 10.4208/aamm.11-m1155
April 2013

Toward a New Algorithm for Nonlinear Fractional Differential Equations

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Received 14 March 2011; Accepted (in revised version) 16 October 2012

Available online 8 March 2013

Abstract. This paper is concerned with the development of an efficient algorithm for the analytic solutions of nonlinear fractional differential equations. The proposed algorithm Laplace homotopy analysis method (LHAM) is a combined form of the Laplace transform method with the homotopy analysis method. The biggest advantage the LHAM has over the existing standard analytical techniques is that it overcomes the difficulty arising in calculating complicated terms. Moreover, the solution procedure is easier, more effective and straightforward. Numerical examples are examined to demonstrate the accuracy and efficiency of the proposed algorithm.

AMS subject classifications: 34A08, 74G10, 35C10, 44A10

Key words: Homotopy analysis method, Laplace transform, fractional differential equations.

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1 Introduction

Fractional calculus is an emerging field and over the last decades it has represented an alternative tool to solve several problems from various fields. Interest in the differentiation and integration of non-integer orders dates back to the nineteenth century. Nowadays, fractional calculus is used to model various phenomena in physics, materials science, control theory, polymer modelling and engineering, such as the rheological behavior of viscoelastic materials, heat transfer and diffusion [1, 2].

The increasing use of fractional differential equations in mathematical models motivates the desire to develop good quality numerical and analytical methods for their solution. Most nonlinear fractional differential equations do not have exact analytic solutions; therefore, approximation and analytical techniques must be used.

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The variational iteration method (VIM) [3, 4], Adomian decomposition method (ADM) [5], homotopy perturbation method (HPM) [6] and homotopy analysis method (HAM) [7–16] are relatively new efficient approaches to provide an analytical approximation to linear and nonlinear problems. In recent years, the application of these methods has been extended to obtain an analytical approximate solution to differential equations of fractional order [3, 4, 17–21].

The ADM and VIM are limited in that the former has complicated algorithms in calculating Adomian polynomials for nonlinear problems, and the latter has an inherent inaccuracy in identifying the Lagrange multiplier for fractional operators, which is necessary for constructing variational iteration formula. The HPM is indeed a special case of the homotopy analysis method [22]. However, mostly, the results given by HPM converge to the corresponding numerical solutions in a rather small region. Although the HAM provides us with a simple way to adjust and control the convergence region of solution series by choosing a proper value for the auxiliary parameter $\hbar$, we face the difficulty in calculating complicated integrals that arise when dealing with strongly nonlinear problems.

Therefore, in this work we will introduce a new alternative procedure to eliminate these disadvantages in solving nonlinear fractional differential equations. The newly developed technique by no means depends on complicated tools from any field. This can be the most important advantage over other methods. It is worth mentioning that the proposed algorithm is an elegant combination of the Laplace transform method and the homotopy analysis method. Some nonlinear fractional differential equations are examined to illustrate the effectiveness, accuracy and convenience of this method, and in all cases, the presented technique performed excellently.

2 Analysis of the new algorithm

The homotopy analysis method (HAM) is a general analytic approach to get series solutions of various types of nonlinear equations. The validity of the HAM is that it provides a simple way to adjust and control the convergence of solution series and provides great freedom to choose proper base functions to approximate a nonlinear problem. Therefore, the HAM can overcome the foregoing restrictions and limitations of perturbation techniques so that it provides a possibility to analyze strongly nonlinear problems.

In this section, we present a modified algorithm of the homotopy analysis method with the help of Laplace transform. This algorithm can be implemented to handle, in a realistic and efficient way, nonlinear fractional differential equations. This new modification improves the accuracy of applying the HAM directly and facilitates the computational work.

To illustrate the basic ideas of the new algorithm, we consider the following nonlinear differential equation of fractional order (more general form can be considered without