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A Fast High Order Iterative Solver for the Electromagnetic Scattering by Open Cavities Filled with the Inhomogeneous Media

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Abstract. The scattering of the open cavity filled with the inhomogeneous media is studied. The problem is discretized with a fourth order finite difference scheme and the immersed interface method, resulting in a linear system of equations with the high order accurate solutions in the whole computational domain. To solve the system of equations, we design an efficient iterative solver, which is based on the fast Fourier transformation, and provides an ideal preconditioner for Krylov subspace method. Numerical experiments demonstrate the capability of the proposed fast high order iterative solver.

AMS subject classifications: 65N06, 78M20

Key words: Helmholtz equation, compact finite difference scheme, discontinuous wave numbers, immerse interface method, fast iterative solver.

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1 Introduction

The scattering properties of open cavities are of high interest to the engineering community, with a number of applications including the design of jet engine inlet ducts and cavity-backed antenna for military and civil use. In this paper we mainly develop a fast high order iterative solver concerning with the electromagnetic scattering from a two-dimensional open cavity filled with inhomogeneous media for large wave number, which is shown in Fig. 1. The ground plane and the walls of the open cavity are assumed as perfect electric conductors (PEC), and the interior of the open cavity is filled

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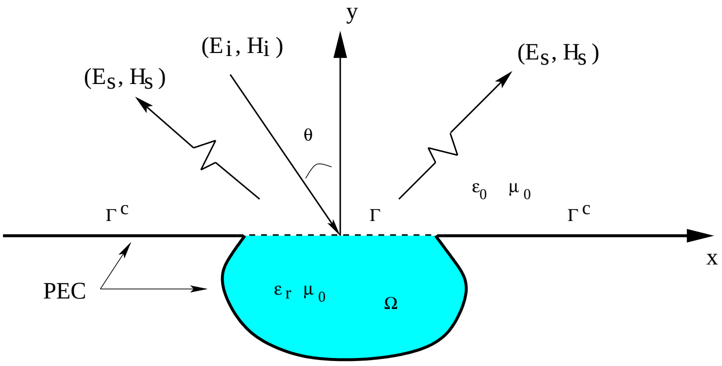

Figure 1: The geometry of the cavity.

with non-magnetic materials which may be inhomogeneous. The half space above the ground plane is filled with a homogenous and isotropic medium with its permittivity ε_0 and permeability μ_0. In this setting, the electromagnetic scattering by the cavity is governed by the Helmholtz equation along with Sommerfeld's radiation conditions imposed at infinity.

Since the solutions to the Helmholtz equation are waves, it is evident that mesh size h must follow the wave number k in order to achieve a given accuracy. For a large wave number, the phase error (pollution) of the computed solution obtained with low order discretization is large unless fine meshes are used per wavelength. See [1] for detailed information. A fine mesh would lead to a large system of equations which may be computationally prohibitive. The memory requirement might be a bottle-neck. Many numerical approaches have been proposed to reduce the phase error. For example, the high order finite element method was proposed in [2]; the h-version and h-p-version finite element methods were proposed in [3,4]. In [5], a standard bilinear finite element together with a modified quadrature rule was used, which led to fourth order phase accuracy on orthogonal uniform meshes. The high order spectral method and compact high order finite difference method have been presented to solve the Helmholtz equation in [6–10,12,13]. In [14], a fully high-order finite element with curvilinear tetrahedral elements was developed to simulate the scattering by cavities. High order methods are attractive for solving the Helmholtz problem with large wave number since they can offer relatively higher accurate solution by utilizing fewer mesh points and spending less computational costs than the low order approaches.

In this paper we first construct a high order finite difference discretization for the scattering of electromagnetic plane waves by a two-dimensional (2-D) rectangular cavity filled with inhomogeneous media. In the cavity domain, the compact fourth order finite difference scheme is used for the discretization of the equation, and at the aperture, a fourth order approximation is also designed by a special technique. In the discontinuous interface of various medium, in terms of the immersed interface method (IIM), see [16, 17], high order accuracy can be obtained. In [19, 20], the related fourth order method