Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **5**, No. 3, pp. 321-336

DOI: 10.4208/aamm.12-m12116
June 2013

Natural Convection in a Concentric Annulus: A Lattice Boltzmann Method Study with Boundary Condition-Enforced Immersed Boundary Method

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Received 14 September 2012; Accepted (in revised version) 19 November 2012

Available online 30 April 2013

Abstract. In this paper, a boundary condition-enforced IBM is introduced into the LBM in order to satisfy the non-slip and temperature boundary conditions, and natural convections in a concentric isothermal annulus between a square outer cylinder and a circular inner cylinder are simulated. The obtained results show that the boundary condition-enforced method gives a better solution for the flow field and the complicated physics of the natural convections in the selected case is correctly captured. The calculated average Nusselt numbers agree well with the previous studies.

AMS subject classifications: 76R10

Key words: Lattice Boltzmann method, boundary condition-enforced immersed boundary method, natural convection.

1 Introduction

Natural convection is a flow process driven by temperature gradient on the gravity condition. Natural convection phenomena exist widely in engineering field, such as air refrigeration, solar energy storage, electronic component cooling and so on. Therefore it has attracted a large amount of attention about the characteristics of flow and heat transfer from researchers. Nowadays there are three basic methods that have been used to study this problem: theoretical analysis, experimental study and numerical simulation. Among them, numerical simulation is popular due to its low cost, efficiency and informativeness. Actually, there are many works to simulate natural convection problems

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using numerical method, especially the natural convection in enclosed spaces. As early as 1983, G Davis published a benchmark solution for natural convection in a square cavity [1]. Later many researchers have studied the natural convection problem in different cases by different methods [2–6].

Recently, a lattice Boltzmann method (LBM), as an alternative method of computational fluid dynamics, has been applied to many areas of fluid dynamics and heat transfer such as natural convection [7–13]. For example, Shu et al. applied LBM to simulate natural convection in a square cavity [7]. Dixit et al. computed the cases of high Rayleigh number natural convection [8]. Peng et al. developed a thermal LBM model to simulate 3D natural convection [10]. Nor Zwadi et al. proposed a Double-Population Thermal LBM to simulate natural convection [11]. Especially, some researchers have studied the natural convection in a concentric annulus using LBM. Peng et al used LBM to simulate natural convection in a concentric annulus between a square outer cylinder and a circular inner cylinder [12]. Shi et al. proposed a finite difference-based LBM to simulate natural convection heat transfer in a horizontal concentric annulus [13].

LBM is a particle-based numerical method at a mesoscopic level. The main advantage of LBM is its simplicity, and easily parallel computing and program coding, moreover setting the boundary condition is simple for LBM. After two decades' development, although the basic theory has been gradually improved, LBM encounters a challenge in simulating fluid problems with complex boundaries because it is based on Cartesian grid. In order to solve this problem, Feng and Michaelides [14] proposed a direct-forcing-boundary-LBM method by introducing Peskin's immersed Boundary method (IBM) [15] into LBM. IBM uses a fixed Eulerian grid in the flow field areas, and set up another set of Lagrangian points to represent objects immersed in the flow field. IBM can be naturally combined with LBM in deal with fluid flows with complex geometries due to both based on Cartesian grid. However, in the early IBM-LBM work [14], the interaction force between fluid and particles is computed by the penalty method which introduces a user-defined spring parameter. To overcome this drawback, Feng and Michaelides [16] later introduced a direct-forcing scheme and Niu et al. [17] proposed a momentum exchange-based IB-LBM for simulation of particles moving in incompressible flow.

Although the direct forcing and momentum-exchange ideas are simple and physically plausible, the non-slip boundary condition is often unable to be satisfied. The direct consequence of them is that streamlines penetrate the immersed boundary. So it is difficult to directly apply in thermal boundary. Recently, Shu et al. developed a boundary condition-enforced IBM to overcome the above drawbacks by solving the Navier-Stokes equations to study the natural convection problem [18]. This method has several advantages such as satisfying the non-slip condition, and introducing a little work to extend the method to thermal boundary including Dirichlet boundary and Neumann boundary. In the present study, the boundary condition-enforced IBM is further introduced to LBM, and the natural convections in a concentric isothermal annulus between a square outer cylinder and a circular inner cylinder are simulated. We use LBM as a fluid field solver and thermal field solver, and make use of the boundary condition-enforced IBM to deal