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A Comparison of the Performance of Limiters for Runge-Kutta Discontinuous Galerkin Methods

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Abstract. Discontinuities usually appear in solutions of nonlinear conservation laws even though the initial condition is smooth, which leads to great difficulty in computing these solutions numerically. The Runge-Kutta discontinuous Galerkin (RKDG) methods are efficient methods for solving nonlinear conservation laws, which are high-order accurate and highly parallelizable, and can be easily used to handle complicated geometries and boundary conditions. An important component of RKDG methods for solving nonlinear conservation laws with strong discontinuities in the solution is a nonlinear limiter, which is applied to detect discontinuities and control spurious oscillations near such discontinuities. Many such limiters have been used in the literature on RKDG methods. A limiter contains two parts, first to identify the "troubled cells", namely, those cells which might need the limiting procedure, then to replace the solution polynomials in those troubled cells by reconstructed polynomials which maintain the original cell averages (conservation). [SIAM J. Sci. Comput., 26 (2005), pp. 995–1013.] focused on discussing the first part of limiters. In this paper, focused on the second part, we will systematically investigate and compare a few different reconstruction strategies with an objective of obtaining the most efficient and reliable reconstruction strategy. This work can help with the choosing of right limiters so one can resolve sharper discontinuities, get better numerical solutions and save the computational cost.

AMS subject classifications: 65M60, 65M99, 35L65

Key words: Limiter, discontinuous Galerkin method, hyperbolic conservation laws.

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1 Introduction

The Runge-Kutta discontinuous Galerkin (RKDG) methods for solving hyperbolic conservation laws are high-order accurate and highly parallelizable methods which can easily handle complicated geometries and boundary conditions. These methods have made their way into the main stream of computational fluid dynamics and other areas of applications. The first DG method was introduced in 1973 by Reed and Hill [15] for the neutron transport problem. A major development of this method was carried out by Cockburn et al. in a series of papers [3–7], in which a framework to solve nonlinear time dependent hyperbolic conservation laws was established. They adopted explicit, nonlinearly stable high order Runge-Kutta time discretizations [18], DG space discretizations with exact or approximate Riemann solvers as interface fluxes and TVB (total variation bounded) nonlinear limiter [17] to achieve nonoscillatory properties, and the method was termed as RKDG method. We will briefly review this method in Section 2. Detailed description of the method as well as its implementation can be found in the review paper [8].

Solutions of nonlinear hyperbolic conservation laws usually have discontinuities even though the initial conditions are smooth, which leads to great difficulty in computing these solutions numerically. An important component of RKDG methods for solving conservation laws with strong shocks in the solution is a nonlinear limiter, which is applied to detect discontinuities and control spurious oscillations near such discontinuities. Many such limiters have been used in the RKDG methods. Cockburn et al. developed the minmod-type TVB limiter [3–7], which is a slope limiter using a technique borrowed from the finite volume methodology. Biswas et al. proposed a moment limiter [1] which is specifically designed for DG methods and works on the moments of the numerical solution. This moment limiter was later improved by Burbeau et al. [2] and improved further by Krivodonova [10]. There are also many limiters developed in the finite volume and finite difference literature, such as various flux limiters [21], monotonicity-preserving (MP) limiters [20] and modified MP limiters [16].

Although there are many limiters that we can use in the RKDG methods, none of them is reported to be obviously better than the others for various problems. Numerical experiments in the literature tell that different limiters usually behave differently for the same problem and the same limiter may behave differently for different problems. There is no guideline for people to choose a right limiter for a certain problem. So a systematic study of limiters is necessary.

Qiu and Shu [14] adopted a new framework to devise a limiter for the RKDG methods. They divided a limiter into two separate parts. The first part is a "troubled-cell indicator", which is a discontinuity detection strategy which detects the cells that are believed to contain a discontinuity and need the limiting procedure. The second part is a solution reconstruction method which is applied only on the detected cells. The troubled-cell indicators can come from any limiters or shock detecting techniques. Focused on the first part of limiters, Qiu and Shu [13] presented an overview of the troubled-cell indicators and made a comparison of their performance in conjunction with a high-order