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Numerical Simulations of Unsteady Flows From Rarefied Transition to Continuum Using Gas-Kinetic Unified Algorithm

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Abstract. Numerical simulations of unsteady gas flows are studied on the basis of Gas-Kinetic Unified Algorithm (GKUA) from rarefied transition to continuum flow regimes. Several typical examples are adopted. An unsteady flow solver is developed by solving the Boltzmann model equations, including the Shakhov model and the Rykov model etc. The Rykov kinetic equation involving the effect of rotational energy can be transformed into two kinetic governing equations with inelastic and elastic collisions by integrating the molecular velocity distribution function with the weight factor on the energy of rotational motion. Then, the reduced velocity distribution functions are devised to further simplify the governing equation for one- and two-dimensional flows. The simultaneous equations are numerically solved by the discrete velocity ordinate (DVO) method in velocity space and the finite-difference schemes in physical space. The time-explicit operator-splitting scheme is constructed, and numerical stability conditions to ascertain the time step are discussed. As the application of the newly developed GKUA, several unsteady varying processes of one- and two-dimensional flows with different Knudsen number are simulated, and the unsteady transport phenomena and rarefied effects are revealed and analyzed. It is validated that the GKUA solver is competent for simulations of unsteady gas dynamics covering various flow regimes.

AMS subject classifications: 82C40, 74H15

Key words: Unsteady flow, covering various flow regimes, kinetic theory of gases, Boltzmann model equation, gas-kinetic unified algorithm, discrete velocity ordinate method, Shakov kinetic model, Rykov kinetic model.

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1 Introduction

How to simulate unsteady aerodynamics covering various flow regimes has been being a difficult and meaningful topic. Studies on unsteady aerodynamics started around 1920 when theoretical analysis became as the primary researching means [1,2]. From then on, theoretical studies on unsteady flow have been high-speedily developed [3,4]. At the same time, the improvements of experimental technologies and combination with theoretical researches have been promoted greatly [5–7]. Computational Fluid Dynamics has become of epochal significance to aerodynamics along with the occurrence of Computer. Then, numerical simulations of unsteady problems [8–11] have been developed from inviscid Euler equation to viscid Navier-Stokes equation, from fixed cell to moving mesh, and from fixed aerofoil to swing wing, and different numerical schemes and computational techniques have been advanced.

On the other hand, researches of unsteady flow problems in rarefied transitional flow regimes have arisen to be a popular subject in recent years [12,13]. It is difficult for classical macro-hydrodynamic equations to describe gas flows in such regimes. Consequently, the numerical methods for unsteady flows of rarefied gases need to be developed [14]. The most widely used one is the direct simulation Monte Carlo (DSMC) method [15] proposed by Bird as early as 1963. The DSMC method represents the massive real gas molecules by a finite set of simulated particles. The spatial coordinates, velocity components and internal energies of these simulated molecules are stored in the computers, and their values change with time because of the movement of simulated molecules, the interactions with the boundary and the collisions between simulated molecules. It eventually achieves the simulation of real gas flow by the statistics of the states of motion from the simulated molecules within the cell. With the developments during the past decades, the DSMC method has been widely applied and validated by rarefied gas dynamics including unsteady flow problems [16–20]. Recently, the DSMC method is adopted for a transient rarefied gas flow through a short tube [21]. A wide range of gas rarefaction is considered, and some important conclusions are given. The results show that it is complicated and difficult to reveal the transient flow phenomena of rarefied gas. Besides, the DSMC method is used to predict the steady and unsteady rarefied gas flows induced by a rotor-stator interaction in a single-stage disk-type drag pump [22]. Actually, statistical noise and fluctuation caused by probabilistic nature of the DSMC is a disadvantageous influence for unsteady flow simulation [23,24].

On the basis of mesoscopic Boltzmann-type velocity distribution function theory connecting macroscopic fluid dynamics and microscopic molecular dynamics [25–27], a new class of methods such as the Discrete Velocity Models (DVM) [27, 28, 61], the Lattice Boltzmann Equation (LBE) [29–32] and the Gas-Kinetic Scheme (GKS) [12,27,33–36] has been being developed for simulation of gas flows covering various regimes. Although these methods are sometimes designed for macroscopic hydrodynamics [31], they are not based upon macroscopic fluid equations like the Navier-Stokes equations; instead, they are closely related to the kinetic theory or the Boltzmann-BGK-type equation. As