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Analysis of Two-Dimensional Thin Structures (From Micro- to Nano-Scales) Using the Singular Boundary Method

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Abstract. This study investigates the applicability of the singular boundary method (SBM), a recent developed meshless boundary collocation method, for the analysis of two-dimensional (2D) thin structural problems. The troublesome nearly-singular kernels, which are crucial in the applications of SBM to thin shapes, are dealt with efficiently by using a non-linear transformation technique. Promising SBM results with only a small number of boundary nodes are obtained for thin structures with the thickness-to-length ratio is as small as 1E-9, which is sufficient for modeling most thin layered coating systems as used in smart materials and micro-electro-mechanical systems. The advantages, disadvantages and potential applications of the proposed method, as compared with the finite element (FEM) and boundary element methods (BEM), are also discussed.

AMS subject classifications: 65N15, 65N38

Key words: Singular boundary method, meshless boundary collocation method, method of fundamental solutions, sinh transformation, thin structural problems.

1 Introduction

The method of fundamental solutions (MFS) [1–3] belongs to the family of meshless boundary collocation methods that present remarkable results with a small computational effort. The MFS is easy-to-implement and computationally efficient and thus a competitive alternative for the solution of boundary value problems. However, the traditional MFS requires a fictitious boundary outside the problem domain in order to avoid

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singularities of the fundamental solutions. Despite many years of hard research, the determination of the fictitious boundary is largely based on experience and presents the most serious drawback in the MFS applications to real-world problems [4–6].

In recent decades, considerable efforts have been made to mitigate this difficulty associated with the traditional MFS, so that the source points can be placed on the real boundary directly. The developed methods include, but are not limit to, the boundary knot method (BKM) [7], regularized meshless method (RMM) [8], modified method of fundamental solutions (MMFS) [9], boundary collocation method (BCM) [10], and boundary distributed source (BDS) method [11]. The merits and drawbacks of the above-mentioned methods over the traditional MFS for solving elliptic boundary value problems are thoroughly discussed in [12, 13].

In a more recent study, a novel meshless boundary collocation method, called the singular boundary method (SBM), was proposed by Chen and his coworkers [12, 14, 15]. The method cures the perplexing fictitious boundary issue associated with the traditional MFS while inheriting the merits of the latter of being truly meshless, integration-free and easy-to-program. The method also offers several advantages over the classical domain or boundary discretization methods. First of all, it is meshless which means that no mesh, but a mere collection of points is required for the discretization of the problem. Second, it does not involve integration which could be otherwise troublesome, expensive, and complicated, as in the case, for example, the boundary element method (BEM). Thirdly, it is a boundary method which means that it shares all the advantages that the BEM has over domain discretization methods such as the finite element method (FEM). Prior to this study, this method has since been successfully applied to a variety of physical problems, such as two- (2D) and three-dimensional (3D) heat-conduction problems [13, 16, 17], infinite domain problems [18], and elasticity problems [12].

This paper is an extension of our previous work [19] where a non-linear coordinate transformation was proposed and applied to treat boundary layered effect occurring in 2D potential problems. Herein, the developed algorithm is extended to the numerical analysis of 2D thin structural problems. The computer program in Fortran 90 is developed for general thin-structural problems. For the test problems studied, very promising results are obtained when the thickness-to-length ratio of the structure is in the orders of 1E-6 to 1E-9, which is sufficient for modeling most thin layered coating systems in the micro- or nano-scales. A brief outline of the rest of this paper is as follows. The SBM formulation and its implementation are presented in Section 2. Section 3 introduces the transformed SBM formulation for 2D thin structural problems. Followed in Section 4, the accuracy and efficiency of the proposed method are tested on two benchmark 2D thin structural problems, in which the proposed method is compared with the FEM and BEM. Finally, some conclusions and remarks are provided in Section 5.

2 The SBM formulation for 2D potential problems