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Numerical Analysis of Damage Thermo-Mechanical Models

Hamdi Hentati[1,*], Ilyes Ben Naceur[2], Wassila Bouzid[2] and Aref Maalej[1]

[1] *LASEM Laboratory, National School of Engineers of Sfax, University of Sfax, Tunisia*
[2] *Unit of Mechanical Production and Materials, National School of Engineers of Sfax, University of Sfax, Tunisia*

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Abstract. In this paper, we present numerical computational methods for solving the fracture problem in brittle and ductile materials with no prior knowledge of the topology of crack path. Moreover, these methods are capable of modeling the crack initiation. We perform numerical simulations of pieces of brittle material based on global approach and taken into account the thermal effect in crack propagation. On the other hand, we propose also a numerical method for solving the fracture problem in a ductile material based on elements deletion method and also using thermo-mechanical behavior and damage laws. In order to achieve the last purpose, we simulate the orthogonal cutting process.

AMS subject classifications: 65M10, 78A48

Key words: Fracture mechanics, brittle and ductile materials, variational approach, Johnson and Cook laws, and orthogonal cutting.

1 Introduction

The behavior of materials can be classified into two categories: brittle and ductile. So, steel and aluminum are usually fall in the class of ductile materials; glass and ceramic are fall in the class of brittle materials. These two categories can be distinguished by comparing the stress-strain curves. The material response for ductile and brittle materials are exhibited by both qualitative and quantitative differences in their respective stress-strain curves. Ductile materials will withstand large strains before the specimen rupture and the rupture in brittle materials fracture occurs at much lower strains. The yielding

*Corresponding author.
Email: hamdi.hentati@yahoo.fr (H. Hentati)

region for ductile materials often takes up the majority of the stress-strain curve, whereas for brittle materials it is nearly nonexistent.

The developments related to the theory of brittle fracture are based on the ideas of Griffith [1]. In this theory, the fundamental quantities are the toughness G_c and the energy release rate G. Propagation will take place if $G \geq G_c$. Despite the important contribution of this theory, it has some shortcomings detailed in [2, 3]. Recently, the fracture mechanics has been revisited by proposing different models of brittle fracture in linearly elastic bodies inspired from the Griffith's criterion. A variational theory was developed by Bourdin et al. [2] end was studied by [3–5] aiming to model brittle fracture. Then, for any displacement and crack configuration, one defines the total energy:

$$E(u,\Gamma) = P(e(u)) + G_c H^{N-1}(\Gamma), \tag{1.1}$$

where $P(e(u))$ denotes the elastic energy of the considered system subject to a displacement u and cracked along Γ. $H^{N-1}(\Gamma)$ denotes the $N-1$ dimensional Hausdorff measure of Γ, i.e., its length in 2D and its surface area in 3D. $e(u)$ is the strain field. In order to simulate the crack propagation based on the variational theory, we employ a numerical method using the alternate minimization algorithm. Therefore, the first objective of our work is to present the mains computational results of the crack initiation and propagation based on the variational approach and using an alternate minimization algorithm. In addition, we examine also the crack formation under thermal shock in brittle material, these numerical results will compared with experimental ones.

On the other hand and in order to propose a numerical method for solving the fracture problem in a ductile material, we simulate the orthogonal cutting of the ductile material by the formation of the discontinuous chip.

Cut modeling is highly conditioned by the relevance of the behavior law which should describe the main phenomena and their interactions. The Johnson and Cook law [6], presented in Eq. (1.2), is used to model orthogonal cutting.

$$\sigma_{eq} = [A + B\varepsilon^n]\Big[1 + C\ln\Big(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\Big)\Big]\Big[1 - \Big(\frac{T - T_{amb}}{T_f - T_{amb}}\Big)^m\Big], \tag{1.2}$$

T_f is the melting temperature, T_{amb} is the room temperature and T is the cutting temperature. A is the yield strength of the work material at room temperature. B and n represent the effects of strain hardening. C is the strain rate constant and m is the thermal softening fraction. The strain rate $\dot{\varepsilon}$ is normalized with a reference strain rate $\dot{\varepsilon}_0 = 1\text{s}^{-1}$. The continuous chip is characterized by a process of plastic deformation in the primary shear zone (see Fig. 1). The chip flows continuously. This configuration is the most modeled in the literature. One used two different methods to simulate the formation of the continuous chip. The first consists in predefining the geometry of the chip [7–9]. The second starts from an arbitrary geometry (i.e., without predefine the geometry of the chip) [10–14].

On the other hand, the discontinuous chip is characterized by its periodic rupture and by the appearance of cracks. The shear stress reaches the limit of the material break