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Solution of the Magnetohydrodynamics Jeffery-Hamel Flow Equations by the Modified Adomian Decomposition Method

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Abstract. In this paper, the nonlinear boundary value problem (BVP) for the Jeffery-Hamel flow equations taking into consideration the magnetohydrodynamics (MHD) effects is solved by using the modified Adomian decomposition method. We first transform the original two-dimensional MHD Jeffery-Hamel problem into an equivalent third-order BVP, then solve by the modified Adomian decomposition method for analytical approximations. Ultimately, the effects of Reynolds number and Hartmann number are discussed.

AMS subject classifications: 76W05, 34B15

Key words: Jeffery-Hamel flow, magnetohydrodynamics, nonlinear differential equation, Adomian decomposition method, Adomian polynomials.

1 Introduction

Jeffery [1] and Hamel [2] have worked on incompressible viscous fluid flow through convergent-divergent channels, mathematically. They presented an exact similarity solution of the Navier-Stokes equations. The special case of two dimensional flow through a channel with inclined plane walls meeting at a vertex and with a source or sink at the vertex has been studied by several authors [3–7].

Most scientific problems such as Jeffery-Hamel flows and other fluid mechanic problems are inherently in form of nonlinearity. Except a limited number of these problems, most of them do not have exact solution. Therefore, these nonlinear equations should be solved using other methods. Therefore, many different methods have been introduced to obtain analytical approximate solutions for these nonlinear problems, such as the perturbation method [8, 9], orthogonal polynomial and wavelet methods [10], methods of

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travelling wave solutions [11], the Adomian decomposition method (ADM) and the variational iteration method [12].

One of the most applicable analytical techniques is the ADM [13–22]. It is a practical technique for solving nonlinear functional equations, including ordinary differential equations, partial differential equations, integral equations, integro-differential equations, etc. The ADM provides efficient algorithms for analytic approximate solutions and numeric simulations for real-world applications in the applied sciences and engineering without unphysical restrictive assumptions such as required by linearization and perturbation. The accuracy of the analytic approximate solutions obtained can be verified by direct substitution.

In the ADM, the solution $u(x)$ is represented by a decomposition series

$$u(x)=\sum_{n=0}^{\infty}u_n(x), \tag{1.1}$$

and the nonlinearity comprises the Adomian polynomials

$$Nu(x)=\sum_{n=0}^{\infty}A_n(x), \tag{1.2}$$

where the Adomian polynomials $A_n(x)$ is defined for the nonlinearity $Nu=f(u)$ as [20]

$$A_n(x)=A_n(u_0,u_1,\cdots,u_n)=\frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}f\Big(\sum_{k=0}^{\infty}\lambda^k u_k(x)\Big)\Big|_{\lambda=0}. \tag{1.3}$$

Different algorithms for the Adomian polynomials have been developed by Rach [23,24], Wazwaz [25], Abdelwahid [26] and several others [27–30]. Recently new algorithms and subroutines in MATHEMATICA for fast generation of the Adomian polynomials to high orders have been developed by Duan [31–33].

The solution components are determined by recursion scheme. The nth-stage approximation is given as $\phi_n(x)=\sum_{k=0}^{n-1}u_k(x)$.

We remark that the convergence of the Adomian series has already been proven by several investigators [23,34–36]. For example, Abdelrazec and Pelinovsky [36] have published a rigorous proof of convergence for the ADM under the aegis of the Cauchy-Kovalevskaya theorem. In point of fact the Adomian decomposition series is found to be a computationally advantageous rearrangement of the Banach-space analog of the Taylor expansion series about the initial solution component function.

2 Mathematical formulation

We present model of the MHD Jeffery-Hamel flow problem, which describes an exact similarity solution of the Navier-Stokes equations for special case of two-dimensional flow through a channel with inclined plane walls.

We will employ cylindrical polar coordinates r, θ to describe the steady two-dimensional flow of an incompressible conducting viscous fluid between two rigid plane walls that meet at an angle 2α ($\alpha>0$). The velocity is assumed to be purely radial and dependent on r and θ only. The governing mathematical equations are [37,38]

$$\frac{\partial}{\partial r}(ru(r,\theta))=0, \tag{2.1a}$$

$$u(r,\theta)\frac{\partial u(r,\theta)}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\Big(\frac{\partial^2 u(r,\theta)}{\partial r^2}+\frac{1}{r}\frac{\partial u(r,\theta)}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u(r,\theta)}{\partial\theta^2}-\frac{u(r,\theta)}{r^2}\Big) -\frac{\sigma B_0^2}{\rho r^2}u(r,\theta), \tag{2.1b}$$

$$\frac{1}{\rho r}\frac{\partial p}{\partial\theta}-\frac{2\nu}{r^2}\frac{\partial u(r,\theta)}{\partial\theta}=0, \tag{2.1c}$$

where ρ represents the fluid density, $u(r,\theta)$ gives the velocity along the radial direction, p is the fluid pressure, ν denotes the coefficient of kinematic viscosity, σ represents the conductivity of the fluid and B_0 is the electromagnetic induction.

The boundary conditions are

$$u(r,\alpha)=0,\quad \frac{\partial u}{\partial\theta}(r,0)=0. \tag{2.2}$$

By virtue of Eq. (2.1a), we introduce

$$f(\theta)=ru(r,\theta), \tag{2.3}$$

from which we calculate

$$\frac{\partial u(r,\theta)}{\partial r}=-\frac{f(\theta)}{r^2},\quad \frac{\partial u^2(r,\theta)}{\partial r^2}=\frac{2f(\theta)}{r^3}.$$

Substituting them into Eqs. (2.1b) and (2.1c), we obtain

$$\frac{1}{\rho}\frac{\partial p}{\partial r}=\frac{f^2(\theta)}{r^3}+\frac{\nu f''(\theta)}{r^3}-\frac{\sigma B_0^2}{\rho r^3}f(\theta), \tag{2.4a}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial\theta}=\frac{2\nu f'(\theta)}{r^2}. \tag{2.4b}$$

From Eqs. (2.4a) and (2.4b) we obtain the third-order differential equation

$$f'''(\theta)+\frac{2}{\nu}f(\theta)f'(\theta)+\Big(4-\frac{\sigma B_0^2}{\rho\nu}\Big)f'(\theta)=0. \tag{2.5}$$

We note that $f(0)>0$ corresponds to a divergent-channel, while $f(0)<0$ corresponds to a convergent-channel.

Introducing the dimensionless variables

$$\eta=\frac{\theta}{\alpha},\quad y(\eta)=\frac{f(\theta)}{f(0)}=\frac{f(\alpha\eta)}{f(0)}, \tag{2.6}$$

we derive the nonlinear BVP for the MHD Jeffery-Hamel problems as

$$y'''(\eta)+2\alpha R_e y(\eta)y'(\eta)+(4-H)\alpha^2 y'(\eta)=0, \tag{2.7a}$$
$$y(0)=1,\quad y'(0)=0,\quad y(1)=0, \tag{2.7b}$$

where $R_e=\alpha f(0)/\nu$ and $H=\sigma B_0^2/\rho\nu$ are the Reynolds and Hartmann numbers, respectively.

3 Solution of the MHD Jeffery-Hamel problems

In Adomian's operator-theoretic notation we write Eq. (2.7a) as

$$Ly(\eta)=Ny(\eta), \tag{3.1}$$

where

$$L(\cdot)=\frac{d^3}{d\eta^3}(\cdot),\quad Ny(\eta)=-(4-H)\alpha^2 y'(\eta)-2\alpha R_e y(\eta)y'(\eta).$$

Taking the inverse linear operator as

$$L^{-1}(\cdot)=\int_0^\eta\int_0^\eta\int_0^\eta(\cdot)d\eta d\eta d\eta, \tag{3.2}$$

then we have

$$L^{-1}Ly(\eta)=\int_0^\eta\int_0^\eta\int_0^\eta y^{(3)}(\eta)d\eta d\eta d\eta=y(\eta)-\Phi(\eta), \tag{3.3}$$

where

$$\Phi(\eta)=y(0)+y'(0)\eta+\frac{\eta^2}{2}y''(0). \tag{3.4}$$

Applying the operator $L^{-1}(\cdot)$ to both sides of Eq. (3.1) yields

$$y(\eta)=\Phi(\eta)+L^{-1}Ny(\eta). \tag{3.5}$$

Using the boundary condition $y(0)=1$ and $y'(0)=0$, we have from Eq. (3.4) as

$$\Phi(\eta)=1+\frac{1}{2}\eta^2 y''(0), \tag{3.6}$$

where $y''(0)$ is an undetermined coefficients. Upon substitution of the formula (3.6) into Eq. (3.5), we obtain

$$y(\eta)=1+\frac{1}{2}\eta^2 y''(0)+L^{-1}Ny(\eta). \tag{3.7}$$

Before we design a modified recursion scheme, we determine the undetermined coefficient $y''(0)$ in advance by algebraic manipulations for the sake of computational advantage [39]. Evaluating $y(\eta)$ at $\eta=1$ and using the boundary condition $y(1)=0$, we have

$$y''(0)=-2-2[L^{-1}Ny(\eta)]_{\eta=1}, \tag{3.8}$$

where the nonlinear integral is

$$[L^{-1}Ny(\eta)]_{\eta=1}=\int_0^1\int_0^\eta\int_0^\eta Ny(\eta)d\eta d\eta d\eta. \tag{3.9}$$

Substituting Eq. (3.8) into Eq. (3.7) we obtain the equivalent nonlinear integral equation for the solution

$$y(\eta)=1-\eta^2-\eta^2[L^{-1}Ny(\eta)]_{\eta=1}+L^{-1}Ny(\eta), \tag{3.10}$$

or equivalently,

$$y(\eta)=1-\eta^2-\eta^2\int_0^1\int_0^\eta\int_0^\eta Ny(\eta)d\eta d\eta d\eta+\int_0^\eta\int_0^\eta\int_0^\eta Ny(\eta)d\eta d\eta d\eta. \tag{3.11}$$

Thus, we have converted the nonlinear BVP into an equivalent nonlinear integral equation without any undermined coefficient.

Next, we substitute the Adomian decomposition series for the solution $y(\eta)$ and the series of the Adomian polynomials for the nonlinearity $Ny(\eta)$ as

$$y(\eta)=\sum_{n=0}^{\infty}y_n(\eta), \quad Ny(\eta)=\sum_{n=0}^{\infty}A_n(\eta), \tag{3.12}$$

where the Adomian polynomials are

$$\begin{aligned}
&A_0(\eta)=-(4-H)\alpha^2y_0'(\eta)-2\alpha R_e y_0(\eta)y_0'(\eta),\\
&A_1(\eta)=-(4-H)\alpha^2y_1'(\eta)-2\alpha R_e(y_0(\eta)y_1'(\eta)+y_1(\eta)y_0'(\eta)),\\
&A_2(\eta)=-(4-H)\alpha^2y_2'(\eta)-2\alpha R_e(y_2(\eta)y_0'(\eta)+y_1(\eta)y_1'(\eta)+y_0(\eta)y_2'(\eta)),\cdots,\\
&A_n(\eta)=-(4-H)\alpha^2y_n'(\eta)-2\alpha R_e\sum_{k=0}^{n}y_k(\eta)y_{n-k}'(\eta).
\end{aligned}$$

Substituting Eq. (3.12) into Eq. (3.10), we have

$$\sum_{n=0}^{\infty}y_n(\eta)=1-\eta^2-\eta^2\Big[L^{-1}\sum_{n=0}^{\infty}A_n(\eta)\Big]_{\eta=1}+L^{-1}\sum_{n=0}^{\infty}A_n(\eta), \tag{3.13}$$

from which we derive the modified recursion scheme

$$y_0(\eta)=1-\eta^2, \tag{3.14a}$$

$$y_{n+1}(\eta)=-\eta^2[L^{-1}A_n(\eta)]_{\eta=1}+L^{-1}A_n(\eta), \quad n\geq 0. \tag{3.14b}$$

By this modified recursion scheme (3.14a), (3.14b) and using the Adomian polynomials, we obtain

$$\begin{aligned}
y_1(\eta) &= \eta^2\Big(-\frac{\alpha^2}{3}+\frac{H\alpha^2}{12}-\frac{2\alpha R_e}{15}\Big)+\eta^4\Big(\frac{\alpha^2}{3}-\frac{H\alpha^2}{12}+\frac{\alpha R_e}{6}\Big)-\frac{1}{30}\alpha\eta^6 R_e,\\
y_2(\eta) &= -\frac{\alpha^2\eta^2\big(315(-4+H)^2\alpha^2-900(-4+H)\alpha R_e+652R_e^2\big)}{75600}\\
&\quad+\eta^4\Big(\frac{1}{144}(-4+H)^2\alpha^4+\frac{1}{40}(4-H)\alpha^3R_e+\frac{1}{45}\alpha^2R_e^2\Big)\\
&\quad+\eta^6\Big(-\frac{1}{360}(-4+H)^2\alpha^4+\frac{1}{60}(-4+H)\alpha^3R_e-\frac{1}{50}\alpha^2R_e^2\Big)\\
&\quad+\eta^8\Big(\frac{1}{280}(4-H)\alpha^3R_e+\frac{1}{140}\alpha^2R_e^2\Big)-\frac{\alpha^2\eta^{10}R_e^2}{1350},\cdots.
\end{aligned}$$

The nth-stage solution approximate is $\phi_n(\eta)=\sum_{k=0}^{n-1}y_k(\eta)$.

Since the exact solution cannot be obtain for the MHD Jeffery-Hamel problems, we consider the error remainder function for the particular nonlinear differential equation $Ly(\eta)-Ny(\eta)=0$,

$$ER_n(\eta)=L\phi_n(\eta)-N\phi_n(\eta)=\frac{d^3\phi_n(\eta)}{d\eta^3}+2\alpha R_e\phi_n(\eta)\frac{d\phi_n(\eta)}{d\eta}+(4-H)\alpha^2\frac{d\phi_n(\eta)}{d\eta}, \tag{3.15}$$

to verify the convergence of our solution and the maximal error remainder parameter

$$MER_n=\max_{0\le\eta\le1}|ER_n(\eta)|, \tag{3.16}$$

which can be conveniently computed by the MATHEMATICA native command "NMaximize" for the nth-stage approximate $\phi_n(\eta)$.

4 Simulation results

The obtained analytical approximations include three parameters: the Reynolds number R_e, the Hartmann number H, and the channel angle α. We present simulation results of the proposed scheme for two Jeffery-Hamel problems, where the Hartmann number H and the Reynolds number R_e are fixed at 0 and 50, respectively. For each problem we investigate the effects of the other two parameters.

Problem 4.1. The Hartmann number H is fixed at 0. We consider the error analytic functions for the two cases: (i) $\alpha=5^\circ$ and $R_e=90$, (ii) $\alpha=9^\circ$ and $R_e=50$. In Figs. 1(a) and 1(b), we plot the error remainder functions $ER_n(\eta)$ for $n=5$ through 8 for the two cases (i) and (ii). The maximal error remainder parameters MER_n for $n=1$ through 12 for the case (i) are listed in Table 1. In Fig. 2, we display the logarithmic plots of the maximal error remainder parameters MER_n versus n for $n=1$ through 12, where the points lie almost

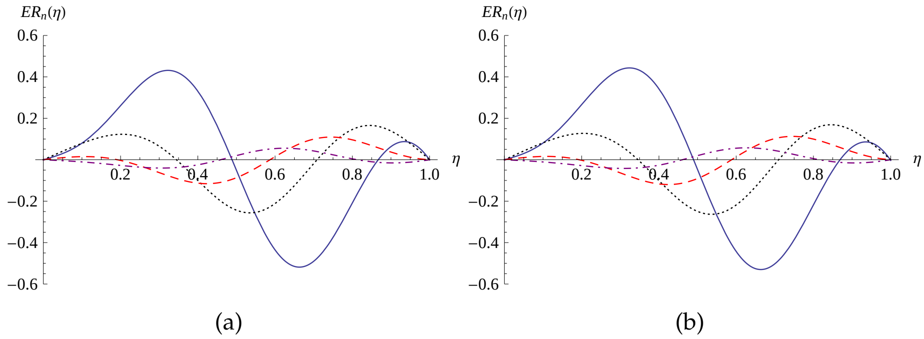

Figure 1: Curves of $ER_n(\eta)$ versus η for $n=5$ (solid line), $n=6$ (dot line), $n=7$ (dash line), $n=8$ (dot-dash line), and for (a) $\alpha=5°$, $R_e=90$, (b) $\alpha=9°, R_e=50$.

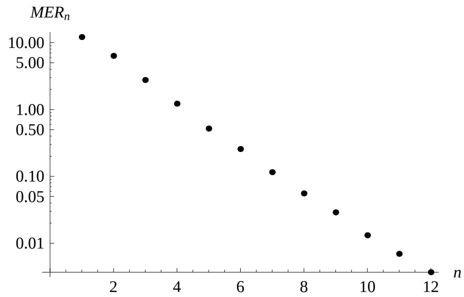

Figure 2: Logarithmic plots of the maximal errors remainder parameters MER_n versus n for $n=1$ through 12.

in a straight line, which indicates that the maximal error remainder parameters decrease approximately at an exponential rate.

In Figs. 3 and 4, we plot the curves of $\phi_{12}(\eta)$ versus η for $\alpha=5°$ and different values of R_e, and for $R_e=50$ and different values of α, respectively. From Figs. 3 and 4, we find that for the fixed channel angle, increase in values of Reynolds number is cause of decreasing in dimensionless velocity, and for the fixed Reynolds number, there is an inverse relation between the channel angle and the velocity of the fluid.

Table 1: The maximal error remainder parameters MER_n for case 1: $H=0$, $\alpha=5°$ and $R_e=90$.

n	1	2	3	4	5	6
MER_n	12.12719	6.34732	2.76374	1.22264	0.51839	0.25637
n	7	8	9	10	11	12
MER_n	0.11555	0.05561	0.028959	0.013163	0.0069443	0.0036860

Problem 4.2. The Reynolds number is fixed at 50. We consider the error analytic functions for the two cases: (i) $\alpha=5°$ and $H=1000$, (ii) $\alpha=9°$ and $H=500$. In Figs. 5(a) and 5(b), we plot the error remainder functions $ER_n(\eta)$ for $n=5$ through 8 for the two cases (i) and (ii). The maximal error remainder parameters MER_n for $n=1$ through 10 for the case (i) are listed in Table 2. In Fig. 6, we display the logarithmic plots of the maximal error

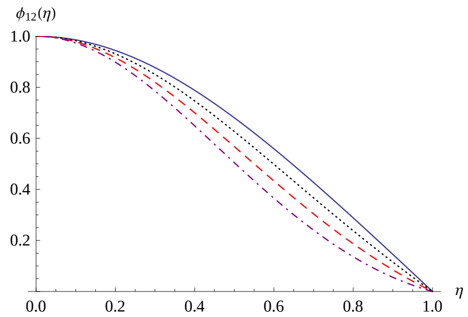

Figure 3: The curves of $\phi_{12}(\eta)$ versus η for $\alpha=5°$ and $R_e=30$ (solid line), $R_e=50$ (dot line), $R_e=70$ (dash line), $R_e=90$ (dot-dash line).

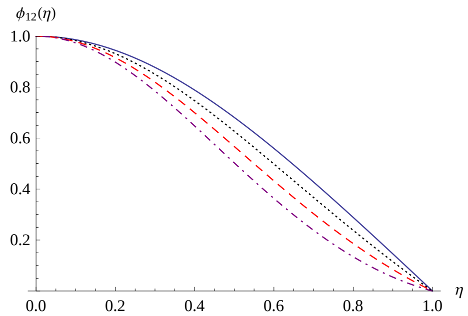

Figure 4: The curves of $\phi_{12}(\eta)$ versus η for $R_e=50$ and $\alpha=3°$ (solid line), $\alpha=5°$ (dot line), $\alpha=7°$ (dash line), $\alpha=9°$ (dot-dash line).

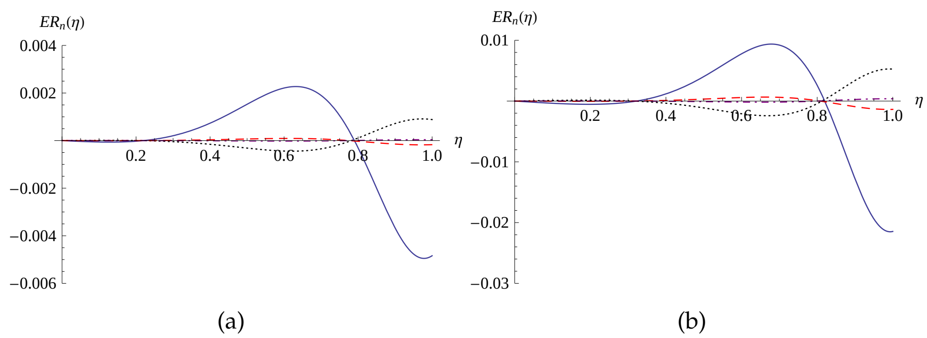

Figure 5: Curves of $ER_n(\eta)$ versus η for $n=5$ (solid line), $n=6$ (dot line), $n=7$ (dash line), $n=8$ (dot-dash line), and for (a) $\alpha=5°$, $H=1000$, (b) $\alpha=9°$, $H=500$.

remainder parameters MER_n versus n for $n=1$ through 10, where the points lie almost in a straight line, which indicates that the maximal error remainder parameters decrease approximately at an exponential rate.

In Fig. 7, we plot the curves of $\phi_{12}(\eta)$ versus η for $\alpha=5°$ and different values of H. For the fixed α, increase in values of Hartmann number is cause of increasing in velocity. The obtained results are consistent with that in [37,38] by using other methods.

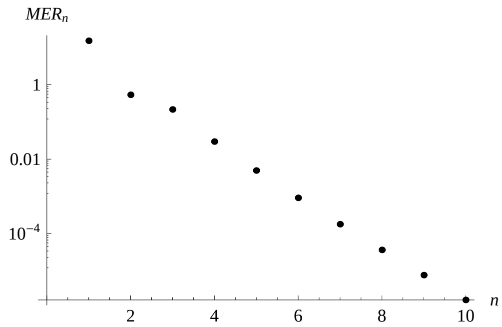

Figure 6: Logarithmic plots of the maximal errors remainder parameters MER_n versus n for $n=1$ through 10.

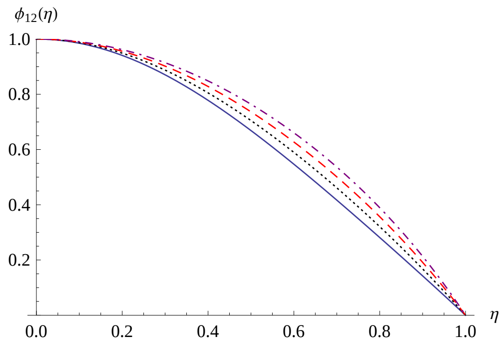

Figure 7: The curves of $\phi_{12}(\eta)$ versus η for $\alpha=5°$ and $H=250$ (solid line), $H=500$ (dot line), $H=750$ (dash line), $H=1000$ (dot-dash line).

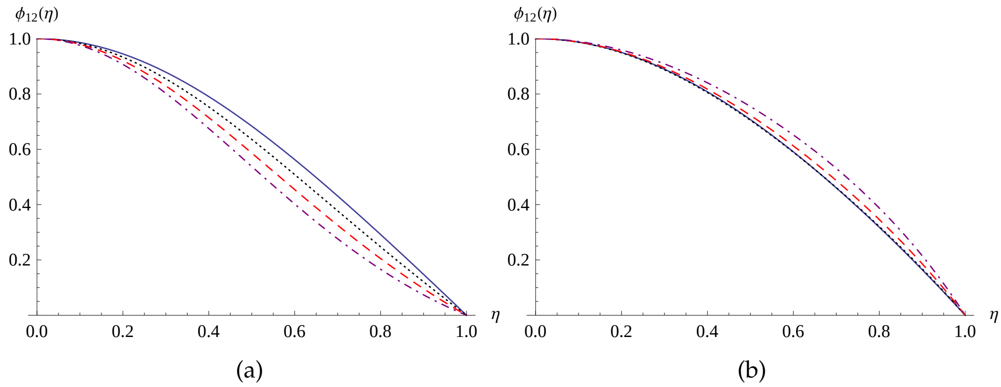

Figure 8: The curves of $\phi_{12}(\eta)$ versus η for $\alpha=3°$ (solid line), $\alpha=5°$ (dot line), $\alpha=7°$ (dash line), $\alpha=9°$ (dot-dash line) and for (a) $H=50$ and (b) $H=500$.

In Figs. 8(a) and 8(b), we plot the curves of $\phi_{12}(\eta)$ versus η for different values of α and for $H=50$ and 500, respectively. For this case, we find that for the fixed Hartmann number $H=50$, increase in values of α is cause of decreasing in velocity. When $H=500$, increase in values of α is cause of increasing in velocity.

Table 2: The maximal error remainder parameters MER_n for case 1: $R_e=50$, $\alpha=5°$ and $H=1000$.

n	1	2	3	4	5
MER_n	15.16995	0.53674	0.21639	0.0296824	0.0049505
n	6	7	8	9	10
MER_n	0.00091182	0.00017847	0.000036405	7.6519×10^{-6}	1.6455×10^{-6}

5 Conclusions

In this research, the modified ADM was applied successfully to find the analytical approximate solutions of the MHD Jeffery-Hamel flows. The results show high accuracy of the method to solve the Jeffery-Hamel problem. Furthermore, the obtained analytical approximations are very convenient for parameter analysis. The effects of the model parameters R_e, H and α on the dimensionless velocity are investigated.

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