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A Switch Function-Based Gas-Kinetic Scheme for Simulation of Inviscid and Viscous Compressible Flows

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Abstract. In this paper, a switch function-based gas-kinetic scheme (SF-GKS) is presented for the simulation of inviscid and viscous compressible flows. With the finite volume discretization, Euler and Navier-Stokes equations are solved and the SF-GKS is applied to evaluate the inviscid flux at cell interface. The viscous flux is obtained by the conventional smooth function approximation. Unlike the traditional gas-kinetic scheme in the calculation of inviscid flux such as Kinetic Flux Vector Splitting (KFVS), the numerical dissipation is controlled with a switch function in the present scheme. That is, the numerical dissipation is only introduced in the region around strong shock waves. As a consequence, the present SF-GKS can well capture strong shock waves and thin boundary layers simultaneously. The present SF-GKS is firstly validated by its application to the inviscid flow problems, including 1-D Euler shock tube, regular shock reflection and double Mach reflection. Then, SF-GKS is extended to solve viscous transonic and hypersonic flow problems. Good agreement between the present results and those in the literature verifies the accuracy and robustness of SF-GKS.

AMS subject classifications: 20B40

Key words: Switch function, gas-kinetic scheme, numerical dissipation, inviscid and viscous flows.

1 Introduction

In the last few decades, the gas-kinetic scheme has been developed systematically and applied in a wide range of flow problems. This approach enjoys a number of advantages, such as robustness, positivity-preserving and satisfying the entropy condition. In

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addition, the "carbuncle phenomenon" can also be avoided in the hypersonic flow simulation, which is quite attractive as compared with conventional Riemann solvers. The gas-kinetic scheme is based on the Boltzmann equation, in which a single gas distribution function is used to describe the entire fluid state. However, due to high dimensionality and complicated collision term, it seems impractical to solve the Boltzmann equation directly except for the simplest problem [1]. On the other hand, the Euler and Navier-Stokes equations are accurate enough in the continuum regime. In fact, the Euler and Navier-Stokes equations can be derived from Boltzmann equation with the Chapman-Enskog expansion analysis. It may be a good choice to combine the Euler/Navier-Stokes solver with the gas-kinetic scheme. That is, with the finite volume discretization, the Euler and Navier-Stokes equations are solved at cell centers and the gas-kinetic scheme is applied to evaluate the numerical flux at cell interfaces.

An early example of gas-kinetic scheme can be referred to the equilibrium flux method (EFM), which was proposed by Pullin [2] for inviscid flows. Mandal and Deshpande [3] pioneered the work of kinetic flux-vector splitting (KFVS) to solve the Euler equations. Furthermore, Chou and Baganoff [4] extended the KFVS scheme for the Navier-Stokes equations. The KFVS has been demonstrated with good positivity property for simulation of flows with strong shock waves [5]. However, as these methods are based on collisionless Boltzmann equation, they usually produce a large numerical viscosity and heat conduction because the numerical dissipation of KFVS is proportional to the mesh size [6]. As a way to reduce the numerical dissipation, Xu [7] proposed the Totally Thermalized Transport (TTT) scheme, where particle collisions are introduced instantaneously in the evaluation of numerical flux at cell interface. The TTT scheme can resolve a boundary layer accurately, but it suffers from oscillations in the region of discontinuity due to insufficient numerical dissipation. It seems that all the above methods cannot capture both the strong shock wave and thin boundary layer at the same time. One of the significant developments in gas-kinetic schemes is the gas-kinetic BGK scheme, which was proposed by Prendergast and Xu [8], Chae et al. [9], Xu [10] and other researchers. In the gas-kinetic BGK scheme, the particle collision effect is taken into consideration in the gas evolution stage and a BGK collision model [11] is applied in the flux evaluation. As a consequence, the dissipation is controlled by the real collision time rather than by the time step. In contrast to conventional upwind schemes, the gas-kinetic BGK scheme computes the inviscid and viscous fluxes simultaneously from the solution of Boltzmann equation with collision term. Numerical results show that this scheme can provide accurate and stable solutions for both inviscid and viscous flow problems [12–20]. However, the gas-kinetic BGK scheme is not completely free from drawbacks. It is usually more complicated and inefficient than conventional CFD schemes. The wish to develop a simple and efficient gas-kinetic scheme to capture both the strong shock wave and thin boundary layer simultaneously motivates the present work.

In the gas-kinetic scheme, the gas distribution function can be expressed as $f = f^{eq} + f^{neq}$, where f^{eq} is the equilibrium distribution function and f^{neq} is the non-equilibrium distribution function. According to Chapman-Enskog analysis, it is known that the equi-