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Sinc Collocation Solutions for the Integral Algebraic Equation of Index-1

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Abstract. In this article, Sinc collocation method is considered to obtain the numerical solution of integral algebraic equation of index-1 by reducing it to an explicit system of algebraic equation. It is shown that Sinc collocation solution can produce an error of order $\mathcal{O}(\sqrt{N}e^{-k\sqrt{N}})$. Moreover, Sinc method is applied to several examples to illustrate the accuracy and implementation of the method.

AMS subject classifications: 65L20

Key words: Integral algebraic equation of index-1, Sinc method, exponential convergence.

1 Introduction

Integral algebraic equations (IAEs) are widely used in mathematical models, for example, [3,5–7]. To describe different features of IAEs, in [4], Gear defined the notion of index. In this paper, we focus on the following IAE of index-1:

$$\begin{cases} y(t)=f_1(t)+\int_0^t K_{1,1}(t,s)y(s)ds+\int_0^t K_{1,2}(t,s)z(s)ds, \\ 0=f_2(t)+\int_0^t K_{2,1}(t,s)y(s)ds+\int_0^t K_{2,2}(t,s)z(s)ds, \end{cases} \tag{1.1}$$

where $K_{i,j}(t,s)$ are defined on $\tilde{D}:=\{(t,s)|t\in\Gamma:=[0,T],\ 0\le s\le t\}$ for $i,j=1,2$, and $(y(t),z(t))$ is a solution to be determined. Particularly, this problem is used in [5] for heat equation with initial and mixed boundary conditions, which represents a boundary reaction in diffusion of chemicals.

For its existence and uniqueness, the conditions are given below.

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Lemma 1.1 (see Theorem 8.1.5 in [2])**.** *Let $v \geq 0$ and assume that*

$$\begin{cases} K_{1,l} \in C^v(\tilde{D}) \ \ for \ \ l=1,2, \\ K_{2,l} \in C^{v+1}(\tilde{D}) \ \ (l=1,2) \ \ and \ \ 0<k_0 \leq |K_{2,2}(t,t)|, \ \ t\in\Gamma, \\ f_1 \in C^v(\Gamma) \ \ and \ \ f_2 \in C^{v+1}(\Gamma) \ \ with \ \ f_2(0)=0. \end{cases} \tag{1.2}$$

Then the Eq. (1.1) *possesses a unique solution $(y(t),z(t))$ on Γ with $y(t),z(t)\in C^v(\Gamma)$.*

Recently, to study IAE (1.1), Kauthen analyzed the spline collocation method and its convergence in [8]. The Legendre collocation method has been proposed for IAE (1.1) in [9] and the posteriori error estimation has been obtained. Meanwhile, Sinc methods have been increasingly recognized as powerful tools for scientific and engineering problems [1, 13, 16]. It is noted from [15] that Sinc methods have some advantages that are worthy of investigation. In fact, for the problems with singularities, Sinc methods are highly efficient and adaptable. It is also well known that Sinc methods are characterized by exponentially decaying errors and do not suffer from the common instability problems for their rapid convergence [17]. For example, Sinc collocation method has been used in [11, 12] to solve the initial and boundary value problems of ordinary and partial differential equations. Especially, in [10], Sinc collocation method was developed for a linear Volterra integral equation of the second kind by using the Sinc basic functions, and it was shown to be efficient and accurate.

In this paper, Sinc collocation method is considered to obtain the numerical solution of (1.1). The layout is as follows. In Section 2, we concern with the Sinc collocation discretization for system of integral equation. Section 3 is concerned with the convergence analysis of the method. Numerical experiments are given in Section 4. Finally, in Section 5, we end with conclusions and future work.

2 The Sinc collocation method

To approximate IAE (1.1), we first recall some concepts and properties of Sinc function in [14].

For all $x \in \mathbb{R}$, the Whittaker's cardinal function of u is defined by

$$C(u,h)(x) = \sum_{j=-\infty}^{\infty} u(jh) S(j,h)(x), \tag{2.1}$$

where the function u is bounded, the step size $h>0$ and j is an integer. Here, the jth translate of Sinc function is defined by

$$S(j,h)(x) = Sinc\Big(\frac{x-jh}{h}\Big),$$