Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **8**, No. 5, pp. 772-783

DOI: 10.4208/aamm.2013.m436
October 2016

Dependence Analysis of the Solutions on the Parameters of Fractional Neutral Delay Differential Equations

Shuiping Yang*

Department of Mathematics, Huizhou University, Guangdong 516007, China

Received 21 December 2013; Accepted (in revised version) 20 July 2015

Abstract. In this paper, we discuss the dependence of the solutions on the parameters (order, initial function, right-hand function) of fractional neutral delay differential equations (FNDDEs). The corresponding theoretical results are given respectively. Furthermore, we present some numerical results that support our theoretical analysis.

AMS subject classifications: 34A08, 34A12, 34K40

Key words: Fractional neutral delay differential equations, Caputo fractional derivative, dependence.

1 Introduction

In this paper, we focus on the following initial value problem (IVPs) of FNDDEs in the form

$$\begin{cases} {}_0^C D_t^\alpha x(t) = f(t, x(t), {}_0^C D_t^\beta x(t-\tau)), & t \in [0,T], \quad 0<\beta<\alpha\le 1, \\ x(t)=\varphi(t), & t\in[-\tau,0], \end{cases} \tag{1.1}$$

where $\tau>0$, $f: D=[0,T]\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}$, and satisfies

$$|I^\alpha[f(t,y,u)-f(t,z,v)]| \le L_1|I^\alpha(y-z)| + L_2|I^\alpha(u-v)|, \tag{1.2}$$

$\forall t\in I=[0,T]$, $\forall y,z,u,v\in C[0,T]$, where L_1, L_2 are positive constants. ${}_0^C D_t^\alpha$ denotes the Caputo fractional derivative of order α and is defined in [5] as

$${}_0^C D_t^\alpha y(t) = I^{1-\alpha}\frac{dy(t)}{dt} = \frac{1}{\Gamma(1-\alpha)}\int_0^t (t-\tau)^{-\alpha}\frac{dy(\tau)}{d\tau}d\tau, \quad t>0, \quad 0<\alpha<1, \tag{1.3}$$

*Corresponding author.
Email: yang52053052@163.com (S. P. Yang)

where I^α denotes the integral operator of order α and is defined in [5] as

$$I^\alpha y(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} y(\tau)d\tau, \quad t>0, \quad \alpha>0.$$

The existence results of (1.1) have been obtained in [11]. The problem (1.1) can be written as

$$\begin{cases} {}_0^{RL}D_t^\alpha(x(t)-x(0)) = f(t,x(t), \ {}_0^{RL}D_t^\beta(x(t-\tau)-x(-\tau))), & t\in[0,T], \quad 0<\beta<\alpha\le 1, \\ x(t)=\varphi(t), & t\in[-\tau,0], \end{cases} \tag{1.4}$$

where ${}_0^{RL}D_t^\alpha$ denotes the Riemann-Liouville fractional derivative of order α and is defined in [5] as

$${}_0^{RL}D_t^\alpha y(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t (t-\tau)^{-\alpha} y(\tau)d\tau, \quad t>0, \quad 0<\alpha<1. \tag{1.5}$$

The relationship between the Caputo definition and the Riemann-Liouville definition can be given by the following formula (cf. [10])

$${}_0^{RL}D_t^\alpha y(t) = {}_0^{C}D_t^\alpha y(t) + \frac{y(0)}{\Gamma(1-\alpha)}t^{-\alpha}, \quad t>0. \tag{1.6}$$

In the recent years, fractional differential equations have been used in various fields such as, fluid-dynamic traffic model, material viscoelastic theory, chemistry and physics (cf. [5–7,10]). In [1,13], some results about the dependence of the solutions on the parameters (including the order of the differential equation, the initial function and the right-hand function) of some classes of fractional differential equations (FDEs) with Riemann-Liouville (R-L) fractional derivatives have been given. Recently, in [12], the authors have extended some results about the dependence in [1] and given the detail analysis of the dependence of the solutions on the above parameters of the IVPs of fractional delay differential equations (FDDEs). In addition, some numerical methods for some classes of fractional neutral differential equations have been discussed in [3,8,9]. In this paper, we will study the dependence of the solutions on these parameters of the IVPs of FNDDEs.

This paper is organized as follows. In Section 2, some theorems about dependence of the solutions on the parameters of FNDDEs are given. Finally, we present some numerical results that support our theoretical analysis.

2 The main results

In this section, we analyze how the solution if influenced under small perturbations of the given parameters, namely, of the initial function φ, of the order of derivative α and the function on the function on the right-hand side of (1.1). Firstly, we introduce the following Lemmas and define the norm

$$\|u(t)\|_\infty = \max_{0\le t\le T}|u(t)|, \quad \forall u(t)\in C[0,T].$$