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Stokes Flow of Viscous Fluid Past a Micropolar Fluid Spheroid

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Abstract. The Stokes axisymmetric flow of an incompressible viscous fluid past a micropolar fluid spheroid whose shape deviates slightly from that of a sphere is studied analytically. The boundary conditions used are the vanishing of the normal velocities, the continuity of the tangential velocities, continuity of shear stresses and spin-vorticity relation at the surface of the spheroid. The hydrodynamic drag force acting on the fluid spheroid is calculated. An exact solution of the problem is obtained to the first order in the small parameter characterizing the deformation. It is observed that due to increase spin parameter value, the drag coefficient decreases. Well known results are deduced and comparisons are made with classical viscous fluid and micropolar fluid.

AMS subject classifications: 76A05, 76D07

Key words: Stokes flow, micropolar fluid, drag force, spheroid, spin vorticity relation.

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1 Introduction

The classical Navier-Stokes theory has proved to be inadequate to describe the behavior of fluids with microstructure such as animal blood, polymeric suspensions, muddy water and lubricants. In the past few years there has been increasing interest in developing theories that can accurately describe the behavior of such fluids. The model of micropolar fluid introduced by Eringen [1,2] represents fluids that exhibit some microscopic effects arising from the local structure and micromotion of the fluid elements and they can sustain couple stresses. These fluids consist of rigid, randomly oriented particles with their own spins and micro rotations suspended in a viscous medium. In the theory of micropolar fluids, there are two vectors describing the motion of the fluid; the

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classical velocity vector and the spin or microrotation vector. The micropolar theory can be applied to increasingly significant number of cases in various fields. Micropolar fluids have been shown to accurately simulate the flow characteristics of polymeric additives, geomorphological sediments, colloidal suspensions, haematological suspensions, liquid crystals, lubricants etc. The review article by Ariman et al. [3] describe some of the applications of miropolar fluids which have been explored. Lukaszewicz [4] has presented the mathematical theory of equations of micropolar fluids and some of its applications.

The problem of motion of one fluid dispersed in another fluid is of special interest because of its applications in various natural and industrial processes, such as raindrop formation, the mechanics and rheology of emulsions, liquid-liquid extraction, and sedimentation phenomena. This type of problems has been studied analytically and numerically by various researchers. The creeping flow motion of a single spherical drop in another immiscible fluid was first analyzed independently by Rybczynski [5] and Hadamard [6]. At the fluid-fluid interface, they have assumed continuity of velocity and tangential shear stress and found the drag force exerted on the fluid sphere by the surrounding fluid. Hetsroni and Haber [7] discussed the problem of a single spherical droplet submerged in an unbounded viscous fluid of a different viscosity. Using spherical bipolar coordinates, Bart [8] examined the motion of a spherical droplet settling normal to a plane interface between two immiscible viscous fluids. Wacholder and Weihs [9] also utilized bipolar coordinates to study the motion of a fluid sphere through another fluid normal to a rigid or free plane surface and their calculations agree with the results obtained by Bart. Lee and Keh [10] employed a combined analytical-numerical method with the boundary collocation technique to examine the quasisteady creeping flow of a spherical drop in an immiscible fluid within a spherical cavity. They obtained the wall-corrected drag force exerted on the drop with good convergence. The Stokes flow of micropolar fluid past a rigid sphere, spheroid and approximate sphere were considered by Rao and Rao [11], Rao and Iyengar [12], and Iyengar and Srinivasacharya [13] respectively. Drag on an axially symmetric body in Stokes flow of a micropolar fluid has been evaluated by Ramkissoon and Majumdar [14] and they observed that the drag in the micropolar fluid is greater than that in the classical fluid. Sawada et al. [15] studied creeping flow of micropolar fluid past a sphere using vector potential.

The cases where both the fluids are Newtonian are discussed above. However, there are cases when one or both the fluids are non-Newtonian in nature. For example the movement of air bubbles in human blood. Ramkissoon [16] has obtained the solution for the problem of a micropolar fluid flow around a Newtonian fluid sphere and evaluated drag force exerted on the sphere. The problem of symmetrical micropolar flow past a Newtonian fluid spheroid whose shape varies slightly from that of a sphere, is examined by Ramkissoon and Majumdar [17]. These two problems are solved by using no-spin microrotation boundary condition. The resistance force exerted on a solid sphere moving with constant velocity in a micropolar fluid with a non-homogeneous boundary condition for the microrotation vector was calculated by Hoffmann et al. [18]. The problem of the flow of a viscous fluid past a micropolar fluid sphere and the flow of a