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On the Computations of Gas-Solid Mixture Two-Phase Flow

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Abstract. This paper presents high-resolution computations of a two-phase gas-solid mixture using a well-defined mathematical model. The HLL Riemann solver is applied to solve the Riemann problem for the model equations. This solution is then employed in the construction of upwind Godunov methods to solve the general initial-boundary value problem for the two-phase gas-solid mixture. Several representative test cases have been carried out and numerical solutions are provided in comparison with existing numerical results. To demonstrate the robustness, effectiveness and capability of these methods, the model results are compared with reference solutions. In addition to that, these results are compared with the results of other simulations carried out for the same set of test cases using other numerical methods available in the literature. The diverse comparisons demonstrate that both the model equations and the numerical methods are clear in mathematical and physical concepts for two-phase fluid flow problems.

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Key words: Hyperbolic conservative equations, two-phase flows, compressible gas-solid, mixture conservation laws, approximate Riemann solver, upwind Godunov methods, numerical simulation.

1 Introduction

For many years the subject of theoretically modelling multi-phase fluid flows has held a prominent place in the attention of applied and computational mathematicians. In more recent years this attention has been directed to the well-defined mathematical models along with numerical methods. The basic issues in the subject traditionally deal with

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hyperbolic or non-hyperbolic character of the governing equations, and conservative or non-conservative character of the governing equations. Typically, two-phase flow seen in a broad range applications and encompasses many different physical processes. It is therefore not possible for a single theoretical framework to describe all the diverse variety of two-phase fluid flow problems. The variety of two phase systems is illustrated by the different mathematical models suggested in the literature. In general, they are usually described by the two-fluid six equations model [4, 5, 11, 30, 31], or the mixture three equations model [14, 19], or the five equations model [13, 15], or the seven equations model [1, 27]. The success of each model depends on the physical phenomena of interest and on the nature of the problem. Almost all these models have non-conservative form causing serious analytical and numerical difficulties. Furthermore, hyperbolicity of such models is obtained under certain restrictions. Recently, an alternative approach based on the theory of thermodynamically compatible systems of hyperbolic conservation laws [7] to model two-phase flows has been proposed (see, for example, [23, 42]). For such an approach, the formulation of thermodynamically compatible systems have been applied to model two-phase gas-liquid [24, 42] and gas-solid mixtures [25, 46] in terms of parameters of state for the mixture. Distinctive features of this approach are that the resulting models admit two pressures, two velocities and two temperatures. Furthermore, the resulting models are fully hyperbolic and fully conservative systems of the governing equations and independent of the kind of numerical method used to implement it. As regard to the numerical tools for the simulation of two-phase flow equations, there are several numerical methods have been proposed in the literature from different perspectives to simulate two-phase flow problems. The details of these numerical methods are very well documented in the literature and not repeated here to which we will refer the reader to the recent papers [2, 6, 10, 12, 17, 18, 29, 34, 37, 40, 44, 47] and references therein for details.

This paper is to continue the present authors investigation in applying advanced numerical methods to solve thermodynamically compatible systems of hyperbolic conservation laws in the context of two-phase fluid flow problems. In a recent study [46], a mathematical model was developed for compressible gas-solid two-phase flow based on the thermodynamically compatible systems theory [7]. Theoretical investigation has shown that the model is fully hyperbolic and fully conservative with a complete mathematical structure of the governing equations. As a consequence, the model equations allows a straightforward application of finite volume methods and corresponding numerical tools. Thus, rather than developing new numerical methods specific to the two-phase flow model of this paper, we propose to adapt a general purpose method for hyperbolic systems of conservation laws that can be applied to the model equations. In earlier studies [23, 24, 42, 43], modern numerical methods such as Godunov methods of centred type were extended to thermodynamically compatible systems of conservation laws in the context of two-phase flow models.

The principal contribution of this paper is to extend and apply the well tested upwind Godunov methods directly to the model of two-phase gas-solid mixture developed in [46]. These methods are usually based on the exact or approximate solution of the local