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Numerical Solution of Fractional Partial Differential Equations by Discrete Adomian Decomposition Method

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Abstract. In this paper we find the solution of linear as well as nonlinear fractional partial differential equations using discrete Adomian decomposition method. Here we develop the discrete Adomian decomposition method to find the solution of fractional discrete diffusion equation, nonlinear fractional discrete Schrodinger equation, fractional discrete Ablowitz-Ladik equation and nonlinear fractional discrete Burger's equation. The obtained solution is verified by comparison with exact solution when $\alpha=1$.

AMS subject classifications: 35R11

Key words: Discrete Adomian decomposition method, Caputo fractional derivative, fractional discrete Schrodinger equation, fractional discrete Burger's equation.

1 Introduction

Fractional differential equations have been the focus on many studies due to their frequent appearance in various fields such as physics, chemistry and engineering. The fractional derivative has been occurring in many physical problems such as frequency dependent damping behavior of materials, motion of a large thin plate in Newtonian fluid, creep and relaxation functions for viscoelastic material, the $PI^{\lambda}D^{\mu}$ controller for the control of dynamical systems etc. Phenomena in electromagnetic, acoustics, viscoelasticity, electrochemistry and material science are also described by differential equations of fractional order [30]. The applicability of this type of equations motivates us to construct efficient methods for solving fractional differential equations. The popular among them are integral transform method [30,32], iterative method [13,19], and Adomian decomposition method [4,5,17].

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Adomian decomposition method is introduced by Adomian and [4,5] has been proven a very useful tool to deal with nonlinear equations. Wazwaz [39] has applied Adomian decomposition method to solve variety of differential equations. While Shawagfeh [34] has employed Adomian decomposition method for solving nonlinear fractional differential equations, Daftardar-Gejji and Jafri have obtained solution of numerous problems [12,25] by using Adomian decomposition method. Also Dhaigude and Birajdar [18] extended the discrete Adomian decomposition method for obtaining the numerical solution of system of fractional partial differential equations.

Recently fractional diffusion equations have attracted attention of many researchers due to its wide applicability both in the theory of mathematical science and technology. They have been used in modelling many physical and chemical processes, heat, mass or electron transfer, pollutants or liquid transport through porous media and engineering problems. Fractional diffusion equations account for typical anomalous feature which is observed in many systems, e.g., the dispersive transports in amorphous semiconductors, porous medium, colloid, proteins, biosystems or even in ecosystems [21, 22, 24, 29]. The recent papers [14, 23, 31] on fractional diffusion equations are valuable in this field. Wyss [38] considered the time fractional diffusion equation and the solution is given in closed form in terms of Fox function. Schneider and Wyss [33], Dhaigude and Nikam [20] considered the time fractional diffusion equation and wave equation and obtained their solutions. Existence and uniqueness of solution of fractional diffusion equations are well studied by Dhaigude [16]. We develop the discrete Adomian decomposition method for fractional discrete diffusion equation.

The nonlinear Schrodinger equation for integer order [15, 27] is a typical dispersive nonlinear partial differential equation that plays a key role in a variety of areas in mathematical physics. Wazwaz employed Adomian decomposition method for solving different types of integer order Schrodinger equation in [36, 37]. Kaya and El-Sayed [26] have solved coupled Schrodinger-KdV equation for integer order using Adomian decomposition method. The Ablowitz-Ladik equation is discovered and studied by Ablowitz-Ladik in [1, 2]. It is the particular case of Schrodinger equation the nonlinear term in the Schrodinger equation is replaced by space discrete form. Bratsos et al. [9] proposed the discrete Adomian decomposition method for the solution of integer order Schrodinger equation. We extend discrete Adomian decomposition method to obtain the numerical solution of nonlinear fractional discrete Schrodinger equation.

In 1915, Bateman [7] studied the Burger's equation which was introduced by Burger [10] in a mathematical modeling of turbulence, and hence it is referred as "one dimensional Burger's equation" which has applications in physics, fluid dynamics, gas dynamics, heat conduction etc. This equation arises in the theory of shock waves, in turbulence problems and in continuous stochastic processes, gas dynamics, heat conduction, elasticity [6, 8]. Abbasbandy and Darvishi [3] developed the Adomian decomposition method for numerical solution of Burger's equation for integer order. Zhu et al. [40] obtained the solution of two dimension integer order Burger's equation by using discrete Adomian decomposition method. We develop the discrete Adomian decomposition method for the solution of nonlinear fractional discrete Burger's equation.

We organize the paper as follows. In Section 2, we define preliminary definitions which are useful for development of our results. Section 3, is devoted for finding the solution of fractional discrete diffusion equation and solution is compare with exact solution when $\alpha=1$. In Section 4 we find the solution of discrete nonlinear fractional Schrodinger equation as well as discrete fractional Ablowitz-Ladik equation (a particular case of discrete fractional Schrodinger equation). In the last section we obtain the solution of nonlinear fractional discrete Burger's equation.

2 Preliminaries and notations

In this section, we set up notations, basic definitions and main properties of Riemann-Liouville integral, and the relation between Riemann-Liouville integral and Caputo fractional derivative is also given.

Definition 2.1. (see [28]) A real function $f(x)$, $x>0$ is said to be in space C_α, $\alpha\in\Re$ if there exists a real number $p>\alpha$ such that $f(x)=x^p f_1(x)$, where $f_1(x)\in C[0,\infty)$.

Definition 2.2. (see [28]) A function $f(x)$, $x>0$ is said to be in space C_α^m, $m\in N\bigcup\{0\}$ if $f^m\in C_\alpha$.

Definition 2.3. (see [30]) Let $f\in C_\alpha$ and $\alpha\geq -1$, then Riemann-Liouville fractional integral of $f(x,t)$ with respect to t of order α is denoted by $J^\alpha f(x,t)$ and is defined as

$$J^\alpha f(x,t)=\frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}f(x,\tau)d\tau, \quad t>0, \quad \alpha>0.$$

The well known property [30] of the Riemann-Liouville operatorJ^α is

$$J^\alpha t^\gamma=\frac{\Gamma(\gamma+1)t^{\gamma+\alpha}}{\Gamma(\gamma+\alpha+1)}.$$

Definition 2.4. (see [11]) For m to be the smallest integer that exceeds $\alpha>0$, the Caputo fractional derivative of $u(x,t)$ with respect to t of order $\alpha>0$ is defined as

$$D_t^\alpha u(x,t)=\frac{\partial^\alpha u(x,t)}{\partial t^\alpha}=\begin{cases} \dfrac{1}{\Gamma(m-\alpha)}\displaystyle\int_0^t (t-\tau)^{m-\alpha-1}\frac{\partial^m u}{\partial t^m}d\tau, & \text{for } m-1<\alpha<m, \\ \dfrac{\partial^m u(x,t)}{\partial t^m}, & \text{for } \alpha=m\in N. \end{cases}$$

Note that the relation between Riemann-Liouville operator and Caputo fractional differential operator is given as follows

$$J^\alpha(D_t^\alpha f(x,t))=J^\alpha(J^{m-\alpha}f^{(m)}(x,t))=J^m f^{(m)}(x,t)=f(x,t)-\sum_{k=0}^{m-1}f^{(k)}(x,0)\frac{t^k}{k!}.$$

3 Fractional discrete diffusion equation

Consider the time fractional space discrete diffusion equation

$$D_t^\alpha u_j(t) = D_h^2 u_j(t) + jhD_h u_j(t) + u_j(t), \quad 0 < \alpha \le 1, \tag{3.1a}$$

$$\text{with initial condition } \; u_j(0) = f_j. \tag{3.1b}$$

It is called discrete initial value problem (IVP). Suppose that $\Delta x = h$ and the function $u(x,t) = u(j\Delta x,t)$ is the discrete function and is denoted by $u_j(t)$. Similarly the function $f(x,0) = f(j\Delta x)$ is the discrete function denoted by f_j. The standard central differences [35] $D_h u_j(t)$ and $D_h^2 u_j(t)$ are defined by

$$D_h u_j(t) = \frac{u_{j+1}(t) - u_{j-1}(t)}{2h}, \quad D_h^2 u_j(t) = \frac{u_{j+1}(t) - 2u_j(t) + u_{j-1}(t)}{h^2}.$$

Note that initial value problem (3.1a)-(3.1b) is the discrete form of initial value problem for diffusion equation

$$D_t^\alpha u(x,t) = Du_{xx}(x,t) + \frac{\partial}{\partial x}(G(x)u(x,t)), \quad 0 < \alpha \le 1, \quad D > 0, \tag{3.2a}$$

$$\text{with initial condition } \; u(x,0) = f(x), \tag{3.2b}$$

where $D_t^\alpha u(x,t)$ is Caputo fractional derivative of order α. In this problem we consider $D = 1$ and $G(x) = x$.

Using Adomian procedure we assume that Eq. (3.1a) has series solution

$$u_j(t) = \sum_{n=0}^{\infty} u_{jn}(t), \tag{3.3}$$

where $u_{jn}(t)$ $(n \ge 0)$ is the approximation of $u_j(t)$. Now, operating J^α on both sides of Eq. (3.1a) and using Eq. (3.3), we get

$$\sum_{n=0}^{\infty} u_{jn}(t) = f_j + J^\alpha \Big(\sum_{n=0}^{\infty} D_h^2 u_{jn}(t) + \sum_{n=0}^{\infty} jhD_h u_{jn}(t) + \sum_{n=0}^{\infty} u_{jn}(t) \Big),$$

where

$$\begin{aligned}
&u_{j0}(t) = f_j, \\
&u_{j1}(t) = J^\alpha \big[D_h^2 u_{j0}(t) + jhD_h u_{j0}(t) + u_{j0}(t) \big], \\
&u_{j2}(t) = J^\alpha \big[D_h^2 u_{j1}(t) + jhD_h u_{j1}(t) + u_{j1}(t) \big], \cdots, \\
&u_{jn+1}(t) = J^\alpha \big[D_h^2 u_{jn}(t) + jhD_h u_{jn}(t) + u_{jn}(t) \big],
\end{aligned}$$

and so on.

Example 3.1. Consider the time fractional diffusion equation

$$D_t^{\alpha}u(x,t)=u_{xx}(x,t)+xu_x(x,t)+u(x,t), \quad 0<\alpha\leq 1, \tag{3.4a}$$

$$\text{with initial condition } \ u(x,0)=x. \tag{3.4b}$$

The discrete form of initial value problem (3.4a)-(3.4b) is

$$D_t^{\alpha}u_j(t)=D_h^2u_j(t)+jhD_hu_j(t)+u_j(t), \quad 0<\alpha\leq 1,$$
$$\text{and initial condition } \ u_j(0)=(jh).$$

By using above procedure, we compute first few approximations

$$u_{j0}(t)=jh, \quad u_{j1}(t)=(jh)\frac{2t^{\alpha}}{\Gamma(\alpha+1)}, \quad u_{j2}(t)=(jh)\frac{2^2t^{2\alpha}}{\Gamma(2\alpha+1)},$$
$$u_{j3}(t)=(jh)\frac{2^3t^{3\alpha}}{\Gamma(3\alpha+1)}, \quad \cdots, \quad u_{jn}(t)=(jh)\frac{2^nt^{n\alpha}}{\Gamma(n\alpha+1)}.$$

On summing, we get

$$u_j(t)=\sum_{n=0}^{\infty}(jh)\frac{2^nt^{n\alpha}}{\Gamma(n\alpha+1)}=(jh)E_{\alpha}(2t^{\alpha}),$$

where E_α is Mittag-Leffler function.

In Fig. 1, we plot the solution $u_j(t)$ for different values of α and show that method has good agreement with the exact solution when $\alpha=1$.

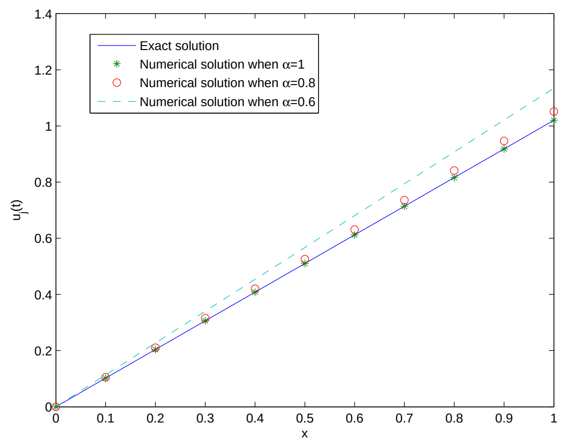


Figure 1: Numerical solution of Example 3.1 when $t=0.01$.

4 Nonlinear fractional discrete Schrodinger equation

The time fractional space discrete nonlinear Schrodinger equation

$$iD_t^\alpha u_j(t)+D_h^2u_j(t)+q|u_j(t)|^2u_j(t)=0,\quad t>0,\quad 0<\alpha\le 1, \tag{4.1a}$$

$$\text{with initial condition}\ \ u_j(0)=f_j, \tag{4.1b}$$

is called discrete initial value problem.

Note that initial value problem (4.1a)-(4.1b) is the discrete form of fractional initial value problem for Schrodinger equation

$$iD_t^\alpha u(x,t)+u_{xx}(x,t)+q|u|^2u=0, \tag{4.2a}$$

$$\text{with initial condition}\ \ u(x,0)=f(x). \tag{4.2b}$$

The Schrodinger Eq. (4.2a) together with initial condition (4.2b) is called initial value problem.

Applying the operator J^α on Eq. (4.1a) and using initial condition, we have

$$u_j(t)=f_j+iJ^\alpha\left(D_h^2u_j(t)+q|u_j(t)|^2u_j(t)\right). \tag{4.3}$$

From the Adomian decomposition method the linear term $u_j(t)$ and the nonlinear term should be decomposed by an infinite series of components such as

$$u_j(t)=\sum_{n=0}^{\infty}u_{jn}(t) \tag{4.4}$$

and

$$|u_j(t)|^2u_j(t)=\sum_{n=0}^{\infty}A_n, \tag{4.5}$$

respectively. Note that $u_{jn}(t)$, $(n\ge 0)$ is the approximation of $u_j(t)$ and those will be elegantly determined whereas A_n $(n\ge 0)$ be Adomian polynomials those can be generated for all forms of nonlinearity. Substituting these decomposition in Eq. (4.3), we obtain

$$\sum_{n=0}^{\infty}u_{jn}(t)=f_j+iJ^\alpha\Big(\sum_{n=0}^{\infty}D_h^2u_{jn}(t)+q\sum_{n=0}^{\infty}A_n\Big). \tag{4.6}$$

Since $u_{j0}(t)$ is identified by the initial data f_j and using the following recurrence relations we find the remaining approximations as follows

$$\begin{aligned}
&u_{j0}(t)=f_j,\\
&u_{j1}(t)=iJ^\alpha\left[D_h^2u_{j0}(t)+qA_0\right],\\
&u_{j2}(t)=iJ^\alpha\left[D_h^2u_{j1}(t)+qA_1\right],\cdots,\\
&u_{jn+1}(t)=iJ^\alpha\left[D_h^2u_{jn}(t)+qA_n\right],\quad n=0,1,2,\cdots,
\end{aligned}$$

where the Adomian polynomials is defined as

$$A_n=\frac{1}{n!}\Big[\frac{d^n}{d\lambda^n}M\Big(\sum_{k=0}^{\infty}\lambda^k u_{jk}(t)\Big)\Big]_{\lambda=0}, \quad n\geq 0. \tag{4.7}$$

Example 4.1. Consider the nonlinear fractional Schrodinger equation

$$iD_t^{\alpha}u(x,t)+u_{xx}(x,t)+q|u(x,t)|^2u(x,t)=0, \quad t>0, \quad 0<\alpha\leq 1, \tag{4.8a}$$

$$\text{with initial condition } u(x,0)=e^{ikx}. \tag{4.8b}$$

The discrete form of initial value problem (4.8a)-(4.8b) is

$$iD_t^{\alpha}u_j(t)+D_h^2u_j(t)+q|u_j(t)|^2u_j(t)=0, j\in Z, \quad t>0, \quad 0<\alpha\leq 1, \tag{4.9a}$$

$$\text{and initial condition } u_j(0)=e^{ijkh}. \tag{4.9b}$$

Here the nonlinear term is $|u_j(t)|^2u_j(t)=u_j^2(t)\bar{u_j}(t)$ which is decomposed as an Adomian polynomials. We compute first few Adomian polynomial for the given nonlinear term as follows

$$\begin{aligned}
A_0=&u_{j0}^2(t)\bar{u_{j0}}(t),\\
A_1=&2u_{j0}(t)u_{j1}(t)\bar{u_{j0}}(t)+u_{j0}^2(t)\bar{u_{j0}}(t),\\
A_2=&2u_{j0}(t)u_{j2}(t)\bar{u_{j0}}(t)+u_{j1}^2(t)\bar{u_{j0}}(t)+2u_{j0}(t)u_{j1}(t)\bar{u_{j1}}(t)+u_{j0}^2(t)\bar{u_{j2}}(t),\\
A_3=&2u_{j0}(t)u_{j3}(t)\bar{u_{j0}}(t)+u_{j1}(t)u_{j2}(t)\bar{u_{j0}}(t)+u_{j0}(t)u_{j2}(t)\bar{u_{j1}}(t)\\
&+u_{j0}(t)u_{j1}(t)\bar{u_{j2}}(t)+u_{j1}^2\bar{u_{j1}}(t)+u_{j0}^2(t)\bar{u_{j3}}(t)+\cdots.
\end{aligned}$$

Here we calculate the components $u_{j1}(t),u_{j2}(t),u_{j3}(t),\cdots$, as

$$\begin{aligned}
u_{j0}(t)&=e^{ijkh},\\
u_{j1}(t)&=iJ^{\alpha}\left[D_h^2u_{j0}(t)+qA_0\right]=-\frac{iwe^{ijkh}t^{\alpha}}{\Gamma(\alpha+1)},\\
u_{j2}(t)&=iJ^{\alpha}\left[D_h^2u_{j1}(t)+qA_1\right]=\frac{-1}{2}w^2e^{ijkh}\frac{t^{2\alpha}}{\Gamma(2\alpha+1)},\\
u_{j3}(t)&=iJ^{\alpha}\left[D_h^2u_{j2}(t)+qA_2\right]=\frac{i}{6}w^3e^{ijkh}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)},
\end{aligned}$$

and so on. Finally summing up the iterates yield

$$u_j(t)=\sum_{n=0}^{\infty}u_{jn}(t)=e^{ijkh}E_{\alpha}(-iwt^{\alpha}).$$

Ablowitz-Ladik equation: The Ablowitz-Ladik equation [1, 2] is the particular case of Schrodinger equation. The nonlinear term in the Eq. (4.1a) is replaced by discrete form as $|u_j(t)|^2(u_{j+1}(t)-u_{j-1}(t))/2$.

Consider the fractional discrete Ablowitz-Ladik equation

$$iD_t^\alpha u_j(t)+D_h^2 u_j(t)+q|u_j(t)|^2\frac{u_{j+1}(t)-u_{j-1}(t)}{2}=0,\quad t>0,\quad 0<\alpha\le 1, \tag{4.10a}$$

$$\text{with initial condition } u_j(0)=f_j, \tag{4.10b}$$

it is called discrete initial value problem. Now, we consider the particular example as follows.

Example 4.2. Consider the fractional discrete Ablowitz-Ladik equation

$$iD_t^\alpha u_j(t)+D_h^2 u_j(t)+q|u_j(t)|^2\frac{u_{j+1}(t)-u_{j-1}(t)}{2}=0 \tag{4.11a}$$

$$\text{and initial condition } u_j(0)=e^{ijkh}. \tag{4.11b}$$

First we calculate the Adomian polynomials for given nonlinear term as follows

$$\begin{aligned}
A_0=&u_{j0}(t)\frac{u_{j+10}(t)+u_{j-10}(t)}{2}\bar{u}_{j0}(t),\\
A_1=&\Big[u_{j0}(t)\frac{u_{j+11}(t)+u_{j-11}(t)}{2}+\frac{u_{j+10}(t)+u_{j-10}(t)}{2}u_{j\bar{1}}(t)\Big]\bar{u}_{j2}(t)\\
&+u_{j0}(t)\frac{u_{j+11}(t)+u_{j-11}(t)}{2}\bar{u}_{j1}(t),\\
A_2=&\Big[u_{j0}(t)\frac{u_{j+12}(t)+u_{j-12}(t)}{2}+\frac{u_{j+10}(t)+u_{j-10}(t)}{2}u_{j2}(t)\Big]\bar{u}_{j0}(t)\\
&+u_{j1}(t)\frac{u_{j+11}(t)+u_{j-11}(t)}{2}\bar{u}_{j0}(t)\\
&+\Big[u_{j0}(t)\frac{u_{j+11}(t)+u_{j-11}(t)}{2}+\frac{u_{j+10}(t)+u_{j-10}(t)}{2}u_{j1}(t)\Big]\bar{u}_{j1}(t)\\
&+u_{j0}(t)\frac{u_{j+10}(t)+u_{j-10}(t)}{2}\bar{u}_{j2}(t).
\end{aligned}$$

Here we find first few approximations as follows:

$$\begin{aligned}
&u_{j0}=f_j=e^{ijkh},\\
&u_{j1}(t)=iJ^\alpha\big[D_h^2u_{j0}(t)+qA_0\big]=\frac{-iwt^\alpha}{\Gamma(\alpha+1)}e^{ijkh},\\
&u_{j2}(t)=iJ^\alpha\big[D_h^2u_{j1}(t)+qA_1\big]=\frac{-1}{2}\frac{w^2t^{2\alpha}}{\Gamma(2\alpha+1)}e^{ijkh},\\
&u_{j3}(t)=iJ^\alpha\big[D_h^2u_{j2}(t)+qA_2\big]=\frac{i}{6}\frac{w^3t^{3\alpha}}{\Gamma(3\alpha+1)}e^{ijkh},
\end{aligned}$$

and so on. Summing all terms we get

$$u_j(t)=\sum_{n=0}^{\infty}u_{jn}(t)=e^{ijkh}E_\alpha(-iwt^\alpha).$$

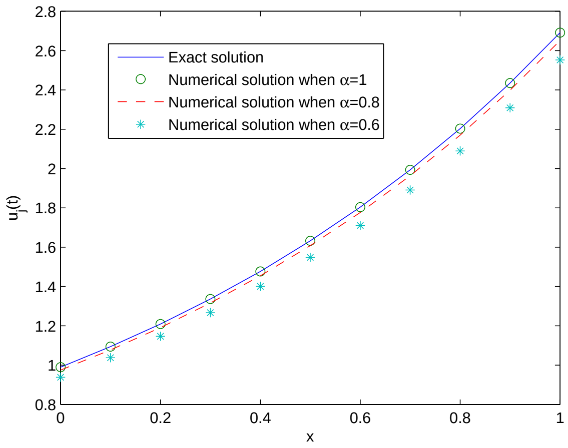


Figure 2: Numerical solution of Example 4.2 when $t=0.01$.

5 Nonlinear fractional discrete Burger's equation

Consider the time fractional space discrete nonlinear Burger's equation

$$D_t^\alpha u_j(t)+u_j(t)D_h u_j(t)=D_h^2 u_j(t) \tag{5.1a}$$

$$\text{with initial condition } \; u_j(0)=f_j. \tag{5.1b}$$

It is called discrete initial value problem.

Note that the initial value problem (5.1a)-(5.1b) is the discrete form of initial value problem for nonlinear fractional Burger's equation

$$D_t^\alpha u(x,t)+u(x,t)u_x(x,t)=u_{xx}(x,t), \quad t>0, \quad x\in\Re, \quad 0<\alpha\le 1 \tag{5.2}$$

with initial condition

$$u(x,0)=f(x). \tag{5.3}$$

Now, operating the operator J^α on Eq. (5.1a) and using initial condition (5.1b), we get

$$u_j(t)=f_j-J^\alpha\left[u_j(t)D_h u_j(t)-D_h^2 u_j(t)\right]. \tag{5.4}$$

The nonlinear operator in Eq. (5.4) can be defined as

$$M(u_j(t))=u_j(t)D_h u_j(t). \tag{5.5}$$

Substituting Eq. (5.5) in Eq. (5.4), we have

$$u_j(t)=f_j-J^\alpha(M(u_j(t))-D_h^2 u_j(t)). \tag{5.6}$$

As per Adomian procedure, the linear terms $u_j(t)$ and the nonlinear term $M(u_j(t))$ should be decomposed by an infinite series of components such as

$$u_j(t)=\sum_{n=0}^{\infty} u_{jn}(t) \tag{5.7}$$

and

$$M(u_j(t)) = \sum_{n=0}^{\infty} A_n,$$

respectively, where A_n is an Adomian polynomials. Using zeroth component the remaining components can be determined by using the recurrence relations as follows:

$$u_{j0}(t) = f_j, \tag{5.8a}$$

$$u_{jn+1}(t) = -J^{\alpha}(A_n) + J^{\alpha}(D_h^2 u_{jn}(t)), \quad n \geq 0. \tag{5.8b}$$

We obtain first few terms of Adomian polynomials A_n which are given as follows:

$$\begin{aligned}
A_0 &= (D_h u_{j0}(t))u_{j0}(t),\\
A_1 &= (D_h u_{j0}(t))u_{j1}(t) + (D_h u_{j1}(t))u_{j0}(t),\\
A_2 &= (D_h u_{j0}(t))u_{j2}(t) + (D_h u_{j1}(t))u_{j1}(t) + (D_h u_{j2}(t))u_{j0}(t),\\
A_3 &= (D_h u_{j0}(t))u_{j3}(t) + (D_h u_{j1}(t))u_{j2}(t) + (D_h u_{j2}(t))u_{j1}(t) + (D_h u_{j3}(t))u_{j0}(t),
\end{aligned}$$

and so on.

By using above recurrence relations we obtain $u_{j1}(t), u_{j2}(t), \cdots$, and its solution is

$$u_j(t) = u_{j0} + u_{j1}(t) + u_{j2}(t) + \cdots.$$

Example 5.1. Consider the nonlinear fractional Burger's equation

$$D_t^{\alpha} u(x,t) + u(x,t)u_x(x,t) = u_{xx}(x,t), \quad t > 0, \quad x \in \Re, \quad 0 < \alpha \leq 1, \tag{5.9a}$$

$$u(x,0) = \sin x. \tag{5.9b}$$

The discrete form of initial value problem (5.9a)-(5.9b) is

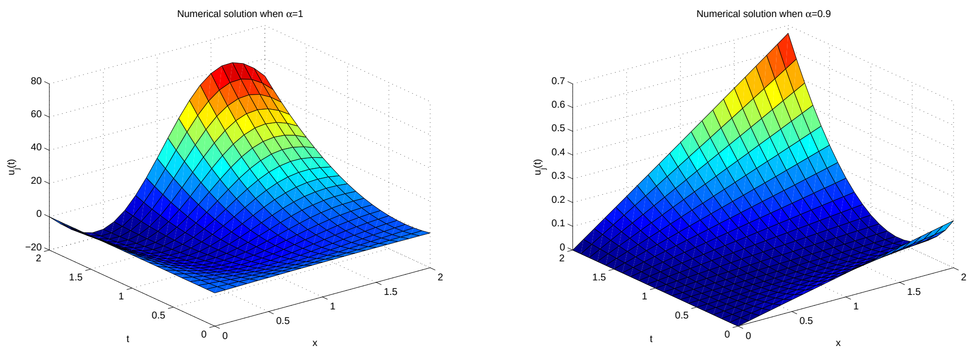
$$\begin{aligned}
&D_t^{\alpha} u_j(t) + u_j(t) D_h u_j(t) = D_h^2 u_j(t)\\
&\text{with initial condition } \ u_j(0) = \sin jh.
\end{aligned}$$

Here we find first few iteration

$$\begin{aligned}
u_{j0}(t) &= \sin(jh),\\
u_{j1}(t) &= \Big[\frac{-1}{2h}\sin(2jh)\sin(h) + v\frac{2(1-\cos(h))}{h^2}\sin(jh)\Big]\frac{t^{\alpha}}{\Gamma(\alpha+1)},\\
u_{j2}(t) &= \Big[\frac{1}{4h}\Big(\frac{\sin(h)}{h}\Big)^2(\sin(3jh) + \sin(jh)) + v\Big(\frac{2(\cos(h)-2)}{h^2}\Big)^2\sin(h)\Big]\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \cdots,
\end{aligned}$$

and so on. The solution of above initial value problem is

$$u_j(t) = u_{j0}(t) + u_{j1}(t) + u_{j2}(t) + \cdots.$$



(a) (b)

Figure 3: Numerical solution of Example 5.1 when $t=0.01$.

6 Conclusions

The discrete Adomian decomposition method is successfully applied to find the solutions of linear as well as nonlinear fractional partial differential equations. The efficiency and accuracy of the proposed method is demonstrated by test problems. It may also be a promising method to solve other nonlinear partial differential equations.

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