Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **6**, No. 2, pp. 191-202

DOI: 10.4208/aamm.2013.m207
April 2014

Asymptotic Study of a Boundary Value Problem Governed by the Elasticity Operator with Nonlinear Term

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Received 30 April 2013; Accepted (in revised version) 18 July 2013

Available online 2 April 2014

Abstract. In this paper, a nonlinear boundary value problem in a three dimensional thin domain with Tresca's friction law is considered. The small change of variable $z=x_3/\varepsilon$ transforms the initial problem posed in the domain Ω^ε into a new problem posed on a fixed domain Ω independent of the parameter ε. As a main result, we obtain some estimates independent of the small parameter. The passage to the limit on ε, permits to prove the results concerning the limit of the weak problem and its uniqueness.

AMS subject classifications: 35R35, 76F10, 78M35

Key words: A priori inequalities, free boundary problems, nonlinear operator, Tresca law, variational problem.

1 Introduction

In this work, we consider a nonlinear boundary value problem governed by partial differential equations which describe the evolution of linear elastic materials in a bounded domain $\Omega^\varepsilon \subset \mathbb{R}^3$ with Tresca's friction law over a portion of the border and Dirichlet boundary conditions on the top and the lateral parts. However, this time we consider a nonlinear term $|u^\varepsilon|^\rho u^\varepsilon$, $\rho=p-2$ for $p>1$. Thus we shall give the analogue of [3], where

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the authors gave the existence and the uniqueness of a non-Newtonian and incompressible fluid with stress tensor $\sigma^{\varepsilon}_{ij}=-p\delta_{ij}+2\mu|D(u^{\varepsilon})|^{r-2}d_{ij}(u^{\varepsilon})$, in a thin domain and the extension of [5, 9]. Before stating the scientific context and our results, we first introduce some notations used in the paper. The boundary Γ^{ε} of the domain is assumed to be Lipschitz continuous so that the unit outward normal n exists almost everywhere on Γ^{ε}. The boundary of the domain is composed of three portions: ω the bottom of the domain, Γ^{ε}_1 the upper surface, and Γ^{ε}_L the lateral surface. Similar studies have been made by several authors but with the usual boundary conditions, we cite for example: In [7], J. L. Lions studied theoretically a problem governed by the Laplace equation with Dirichlet boundary conditions. He proved the existence of a solution based essentially on the method of compactness, and the uniqueness of the solution by imposing conditions on the data. In [9], the authors, studied the similar nonlinear hyperbolic boundary value problem governed by partial differential equations which describe the evolution of the linear elastic materials but with Dirichlet-Neumann usual boundary conditions. They used the techniques of [7, 8] for a particular problem by replacing the elasticity equation by the Laplace operator and with the Neumann boundary conditions. The study of the asymptotic analysis of the same problem but in the particular case where $\rho=1$ has been considered in [5]. In the last few years, some research papers have been written dealing with the asymptotic analysis of an incompressible fluid in a three-dimensional thin domain, when one dimension of the fluid domain tends to zero, (see e.g., [1, 3, 4]) and the references cited therein. More recently, the authors in [2] have studied the asymptotic analysis of a dynamical problem of isothermal elasticity with non linear friction of Tresca type but without the intervention of the nonlinear term. In [10] they studied the asymptotic behaviour of a dynamical problem of non-isothermal elasticity materials. The paper is structured as follows. In Section 2 we present some notations and give the problem statement and variational formulation. In Section 3 we use the asymptotic analysis, in which the small parameter is the height of the domain. We establish some estimates, independent on the parameter ε. These estimates will be useful in order to prove the convergence of the displacement toward the expected function. In Section 4, we investigate the convergence results of the limit weak problem and its uniqueness.

2 Problem statement and variational formulation

Let ω be a fixed bounded domain of $\mathbb{R}^3$ of equation $x_3=0$. We suppose that ω has a Lipschitz continuous boundary and is the bottom of the domain. The upper surface $\overline{\Gamma}^{\varepsilon}_1$ is defined by $x_3=\varepsilon h(x)=\varepsilon h(x_1,x_2)$. We introduce a small parameter ε, that will tend to zero, and a function h on the closure of ω such that $0<h_{\min}\le h(x)\le h_{\max}$, for all $(x,0)$ in ω. We study the asymptotic behaviour of an elasticity in the domain:

$$\Omega^{\varepsilon}=\big\{(x,\,x_3)\in\mathbb{R}^3:(x,0)\in\omega,\ \ 0<x_3<\varepsilon h(x)\big\},$$

and Γ^{ε} its boundary : $\Gamma^{\varepsilon}=\overline{\Gamma}^{\varepsilon}_1\cup\overline{\Gamma}^{\varepsilon}_L\cup\overline{\omega}$, where $\overline{\Gamma}^{\varepsilon}_L$ is the lateral boundary.