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Two-Grid Discretization Scheme for Nonlinear Reaction Diffusion Equation by Mixed Finite Element Methods

Luoping Chen[1] and Yanping Chen[2,*]

[1] *School of Mathematic, Southwest Jiaotong University, Chengdu 611756, P.R. China*
[2] *School of Mathematic science, South China Normal University, Guangzhou 510631, P.R. China*

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Abstract. In this paper, we study an efficient scheme for nonlinear reaction-diffusion equations discretized by mixed finite element methods. We mainly concern the case when pressure coefficients and source terms are nonlinear. To linearize the nonlinear mixed equations, we use the two-grid algorithm. We first solve the nonlinear equations on the coarse grid, then, on the fine mesh, we solve a linearized problem using Newton iteration once. It is shown that the algorithm can achieve asymptotically optimal approximation as long as the mesh sizes satisfy $H=\mathcal{O}(h^{\frac{1}{2}})$. As a result, solving such a large class of nonlinear equations will not be much more difficult than getting solutions of one linearized system.

AMS subject classifications: 65M12, 65M15, 65M60

Key words: Two-grid method, reaction-diffusion equations, mixed finite element methods.

1 Introduction

In this paper, we study the following nonlinear reaction-diffusion equations:

$$c(p)\frac{\partial p}{\partial t}-\nabla\cdot(K\nabla p)=f(p), \qquad (x,t)\in\Omega\times J, \tag{1.1}$$

with initial condition

$$p(x,0)=p^0(x), \qquad x\in\Omega, \tag{1.2}$$

*Corresponding author.
Email: clpchenluoping@163.com (L. Chen), yanpingchen@scnu.edu.cn (Y. Chen)

and boundary condition

$$K\nabla p\cdot\boldsymbol{\nu}=0, \qquad (x,t)\in\partial\Omega\times J, \tag{1.3}$$

where $\Omega\in\mathbb{R}^2$ is a bounded and convex domain with C^1 boundary $\partial\Omega$, $\boldsymbol{\nu}$ is the unit exterior normal direction to $\partial\Omega$, $J=(0,T]$, K: $\Omega\times\mathbb{R}\to\mathbb{R}^{2\times 2}$ is a symmetric positive definite tensor. (1.1) can be rewritten as followings

$$c(p)\frac{\partial p}{\partial t}+\nabla\cdot\boldsymbol{u}=f(p), \tag{1.4}$$

$$K^{-1}\boldsymbol{u}+\nabla p=0. \tag{1.5}$$

Two-grid method is based on the fact that the non-symmetry, indefiniteness and nonlinearity behaving like low frequencies are governed by the coarse grid and the related high frequencies are governed by some linear symmetric positive-definition operators. It was first proposed by Xu [18, 19] as a discretized method for non-symmetric, indefinite and nonlinear partial differential equations. The basic procedure of the two-grid method is to solve a complicated problem (non-symmetric indefinite or nonlinear, etc.) on the coarse grid and then solve a simple symmetric positive or linearized problem on the fine mesh. Because of its simplicity and efficiency, there are lots of investigation of two-grid method for different types of equations in the past few decades. For instance, Chen and Huang studied a multilevel iterative method for solving the finite element solutions of nonlinear singular two-point boundary value problems [4]. Xu and Zhou discussed the algorithm for eigenvalue problem [20]. Zhong [21] analyzed it for Maxwell equations and Layton [12] concerned the scheme for MHD system.

Reaction-diffusion equations have received a great deal of attention motivated by their widespread occurrence in models of hydrologic, biology and bio-geochemical phenomena [11, 13]. Classic examples include the modeling of groundwater through porous media [7]. In this case, p denotes the fluid pressure, $\boldsymbol{u}$ is the Darcy velocity of the flow and $f(p)$ models the external flow rate. Here, for brevity, we drop the dependence of variable x in $f(x,p)$.

Mixed finite element methods have been found to be very important for solving parabolic partial differential equations [10, 14]. For example, there are many applications of mixed finite element methods to miscible displacement problems that describe two-phase flow in petroleum reservoir [7]. Mixed methods have played a fundamental role in discretizing fluid dynamic problems since both the pressure and the flux, or displacements and stresses, are approximated simultaneously.

For nonlinear parabolic equations, two-grid methods were first applied to mixed finite element method by Dawson and Wheeler with f dependent on p, ∇p [8]. Later, they concerned the equations with nonlinear diffusion coefficients by two-grid difference method [9]. Moreover, Wu and Allen [17] established and analyzed a two-step two-grid algorithm for nonlinear reaction-diffusion equations discretized by expanded mixed finite element method. Based on these work, we proposed a three-step two-grid