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Mixed Convection in Viscoelastic Boundary Layer Flow and Heat Transfer Over a Stretching Sheet

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Abstract. An investigation is carried out on mixed convection boundary layer flow of an incompressible and electrically conducting viscoelastic fluid over a linearly stretching surface in which the heat transfer includes the effects of viscous dissipation, elastic deformation, thermal radiation, and non-uniform heat source/sink for two general types of non-isothermal boundary conditions. The governing partial differential equations for the fluid flow and temperature are reduced to a nonlinear system of ordinary differential equations which are solved analytically using the homotopy analysis method (HAM). Graphical and numerical demonstrations of the convergence of the HAM solutions are provided, and the effects of various parameters on the skin friction coefficient and wall heat transfer are tabulated. In addition it is demonstrated that previously reported solutions of the thermal energy equation given in [1] do not converge at the boundary, and therefore, the boundary derivatives reported are not correct.

AMS subject classifications: 76D10, 76W05

Key words: Viscoelastic fluid, non-uniform heat source/sink, viscous dissipation, thermal radiation, homotopy analysis method.

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1 Introduction

More than 100 years after Blasius equation was formulated to describe the boundary layer present in uniform viscous flow over a semi-infinite plate [2], researchers in engineering and applied mathematics continue to investigate the nonlinear differential equations that describe boundary layer flow. Since the landmark work of Blasius, variations of the classical problem have been formulated that consider different flow scenarios and incorporate relevant physical phenomena. In practically all cases considered, the differential equations governing the flow are nonlinear, and the existence of exact solutions is

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rare. As such numerous analytical methods–one of the earliest being the classical perturbation methods–have been developed over the years in order to find approximate solutions. Singular perturbation theory, which is appropriate in the analysis of boundary layer flow, has had a profound impact in the applied sciences and, in particular, quantum physics as evidenced by its vast use over the past 100 years.

As successful as the perturbation methods have been, they have the drawback of relying on the existence of a small or large parameter to be valid. Because of this drawback, various alternative analytical techniques have been developed over the past several decades that do not rely on the existence of a small or large parameter. This paper highlights one of these techniques, namely the homotopy analysis method (HAM) [3–10], which has proven to be a valuable tool in solving not only the nonlinear differential equations of fluid mechanics but also in solving numerous other problems arising in engineering, finance, and the applied sciences. In comparison to other analytical methods, HAM offers the ability to adjust and control the convergence of a solution via the so-called convergence-control parameter.

In a study of two-dimensional boundary layer flow over a moving surface in a fluid at rest, Sakiadis [11] demonstrated that the flow was governed by Blasius equation with different boundary conditions than the Blasius flow. The results of this study were later extended by Crane [12] to include an exact analytical solution for the case of a linearly stretching sheet. It is worth noting that both of these studies consider a Newtonian fluid in the analysis. In recent years, research on the fluid dynamics and heat transfer of boundary layer flow involving non-Newtonian fluids has received increased attention due their growing importance in numerous industrial and biomedical applications. The mechanical properties of the products involved in these applications can be substantially altered by the rate of stretching, rate of cooling, application of a magnetic field, etc., and so, understanding the viscous and thermal characteristics of non-Newtonian fluids is paramount.

Of particular interest is a subclass of non-Newtonian fluids called viscoelastic fluids. Vajravelu and Rollins [13] investigated the fluid flow and heat transfer of a second-order fluid over a stretching sheet with viscous dissipation and internal heat source/sink. Sarma and Rao [14] extended these results by including the effects of work due to elastic deformation, noting that the exclusion of this effect is not in accordance with the inclusion of viscous dissipation. Pillai et al. [15] provided a similar analysis for the flow of a Walters' liquid B fluid in a porous medium. Abel et al. [16] considered the effects of viscous dissipation and a non-uniform heat source/sink on the flow and heat transfer characteristics of a Walters' liquid B fluid. Arnold et al. [17] and Nandeppanavar et al. [1] followed with similar studies by incorporating work due to deformation.

In various engineering processes, it is well understood that thermal radiation plays a significant role when operating temperatures are high. For example in the design of advanced energy conversion systems [18], effects of thermal radiation on the flow and heat transfer characteristics can be quite significant. Raptis and Perdikis [19] were the first to study the effects of thermal radiation on viscoelastic boundary layer flow. A recent study