

Magnetoelastic Instability of a Long Graphene Nano-Ribbon Carrying Electric Current

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Abstract. This paper aims at investigating the resonance frequencies and stability of a long Graphene Nano-Ribbon (GNR) carrying electric current. The governing equation of motion is obtained based on the Euler-Bernoulli beam model along with Hamilton's principle. The transverse force distribution on the GNR due to the interaction of the electric current with its own magnetic field is determined by the Biot-Savart and Lorentz force laws. Using Galerkin's method, the governing equation is solved and the effect of current strength and dimensions of the GNR on the stability and resonance frequencies are investigated.

AMS subject classifications: 82D80, 37N15, 74H55

Key words: Graphene nano-ribbon, resonance frequency, current carrying, Hamilton's principle, Biot-Savart, Lorenz force.

1 Introduction

Recent progresses in nanotechnology have led to the development of nano-electro-mechanical systems (NEMS). Carbon nanostructures such as nanotubes, nanocones and graphene nanoribbons are widely used as nanosensors, nanomechanical resonators, nanoswitches robotic manipulators and magneto-elastic biosensors. The very high stiffness, low density, specific optical properties, high current carrying capability and having two-dimensional structure, have attracted the attention of scientists to GNRs [1–4]. Owing to these outstanding properties, graphene is an ideal material for the design and

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development of new NEMS for a variety of applications, including force, position and mass sensing [5–9].

The structural instability is one of the major problems encountered in flexible lightweight components of NEMS. The vibration and instability of a current-carrying elastic rods have been studied by some researchers [10–13]. Also, recently Chen and et al. [14] investigated the fabrication and electrical readout of monolayer Graphene resonators, and studied their response to changes in mass and temperature.

The aim of this study is to investigate the resonance frequencies and instability of a long GNR carrying electric current. The Lorentz force produced by the interaction of the current with its own magnetic field induces the transverse deflection of GNR. The GNR is modeled as an Euler-Bernoulli beam and the Galerkin method is applied to solve the governing equation of motion. Based on the obtained model, the variation of resonance frequencies and instability conditions of the GNRs of different dimensions are investigated.

2 Governing equations of motion

Fig. 1 shows a schematic of a GNR of flexural rigidity D , length l , width b and thickness h which carries electric current I . The transverse vibration of the GNR is described in the global xyz frame so that the x axis coincides with the neutral axis. The GNR is suspended across a valley between two metallic gates, and is bridge at both ends. Considering the GNR as an Euler-Bernoulli beam, the governing equation of transverse deflection can be derived using Hamilton's principle;

$$\delta H = \int_{t_1}^{t_2} \delta(K - U + W) dt = 0, \quad (2.1)$$

where K is the kinetic energy, U is the potential energy, and W is the work done by the self induced Lorentz force. The kinetic and potential energies of the beam are given by

$$K = \frac{1}{2} \int_0^l \rho A \dot{w}^2 dx, \quad (2.2a)$$

$$U = \frac{1}{2} \int_0^l D w''^2 dx, \quad (2.2b)$$

where ρA is mass of GNR per unit length and w is the transversal deflection. Also the prime and dot symbols denote the derivative with respect to x and time, respectively. The work done by a transverse force distribution f_y on the GNR is calculated as

$$W = \int_0^l f_y w dx. \quad (2.3)$$

The GNR can be modeled as a series of differential segments as shown in Fig. 2. The magnetic field due to the element dx_1 on the neutral axis at point x is obtained from the

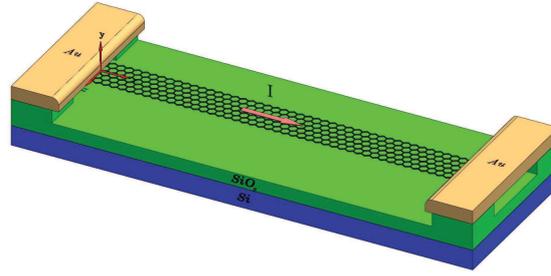


Figure 1: Schematic of the flexible Graphene nano-ribbon.

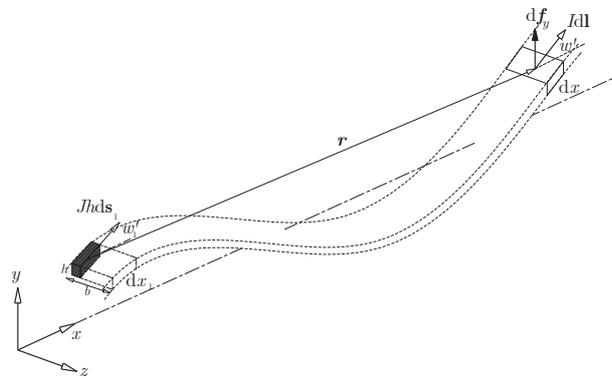


Figure 2: Schematic of the deform GNR and the Lorentz force due to the magnetic field of the element dx_1 on the element dx .

Biot-Savart law as follows [15]

$$\mathbf{B} = \frac{\mu_0}{4\pi} \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{d\mathbf{s}_1 \times \mathbf{r}}{|\mathbf{r}|^3}, \tag{2.4}$$

where $\mathbf{r} = (x - x_1)\mathbf{i} + (w - w_1)\mathbf{j} - z_1\mathbf{k}$ is the distance vector from the element $dx_1 dz_1$ to the neutral axis at point x . Also, μ_0 is the permeability of free space and the current vector $d\mathbf{s}_1$ can be expressed as

$$d\mathbf{s}_1 = Jh(\mathbf{i} + w'_1\mathbf{j})dx_1 dz_1, \tag{2.5}$$

where J is the electric current density. Thus the Lorentz force due to the magnetic field of the element dx_1 on the element dx can be obtained as follows

$$d\mathbf{f} = I d\mathbf{l} \times \mathbf{B}, \tag{2.6}$$

where $d\mathbf{l}$ is $(\mathbf{i} + w'\mathbf{j})dx$. Substituting Eq. (2.4) into Eq. (2.6), the transverse force distribu-

tion f_y on the element dx can be shown

$$f_y = \frac{d\mathbf{f} \cdot \mathbf{j}}{dx} = \int_0^l \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{\mu_0 b h^2 j^2}{4\pi} \frac{w_1 + (x-x_1)w'_1 - w}{((x-x_1)^2 + (w-w_1)^2 + z_1^2)^{3/2}} dz_1 dx_1. \quad (2.7)$$

Assuming constant electric current density across the GNR's width, integrating Eq. (2.8) over the GNR's width and using the Taylor expansion about $w-w_1$, the transverse force distribution is obtained

$$f_y = \frac{\mu_0 I^2}{4\pi} \int_0^l \frac{w_1 + (x-x_1)w'_1 - w}{(x-x_1)^2 \sqrt{(x-x_1)^2 + \frac{b^2}{4}}} dx_1. \quad (2.8)$$

In which the nonlinear terms are eliminated.

3 Solution method

Based on Galerkin's method, the transverse deflections of the GNR can be written as series expansions of the mode shapes of a GNR bridged at both ends, namely

$$w(x,t) = \sum_{n=1}^N \alpha_n(t) \phi_n(x), \quad (3.1)$$

where $\alpha_n(t)$ s are the modal coordinates of the system and ϕ_n s are the modes of a bridged beam, given by

$$\phi_n(x) = \sqrt{\frac{2}{l}} \sin\left(\frac{n\pi x}{l}\right). \quad (3.2)$$

Substituting Eq. (3.1) into Eq. (2.1), and choosing ϕ_m , $m=1, \dots, N$ as admissible variations of the transverse deflection, δw , the following coupled system equations is obtained

$$\mathbf{I}\ddot{\alpha} + \mathbf{K}\alpha = \mathbf{0}, \quad (3.3)$$

where \mathbf{I} is the identity matrix and the stiffness matrix \mathbf{K} is defined

$$\mathbf{K}_{(m,n)} = \frac{D}{\rho A} \left(\frac{n\pi}{l}\right)^4 \delta_{mn} - \frac{\mu_0 I^2}{4\pi \rho A} \int_0^l \int_0^l \frac{\phi_n(x_1) - \phi_n(x) + (x-x_1)\phi'_n(x)}{(x-x_1)^2 \sqrt{(x-x_1)^2 + \frac{b^2}{4}}} \phi_m(x) dx_1 dx, \quad (3.4)$$

where δ_{mn} is Kronecker's delta. Assuming a solution of the form $\mathbf{q} = e^{\lambda_j t} \bar{\mathbf{q}}_j$, where $\lambda_j = \sigma_j + i\omega_j$ is the j^{th} eigenvalue and $\bar{\mathbf{q}}_j$ is the j^{th} eigenvector. Eq. (3.3) is transformed into the following standard eigenvalue problem

$$\mathbf{K}\bar{\mathbf{q}}_j = -\lambda_j^2 \mathbf{I}\bar{\mathbf{q}}_j. \quad (3.5)$$

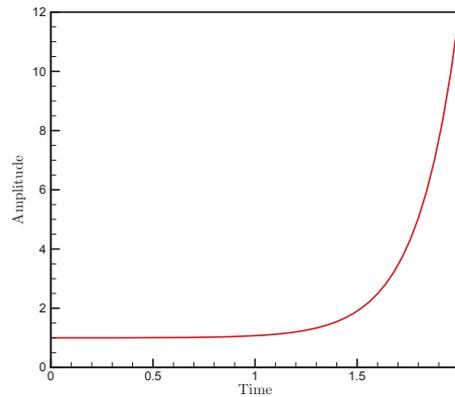


Figure 3: Time response versus amplitude of unstable mode.

To find the nontrivial solutions, the determinant of the matrix $(\lambda_j^2 \mathbf{I} + \mathbf{K})$ must be zero. Thus, the characteristic equation of the system is obtained as

$$|\lambda_j^2 \mathbf{I} + \mathbf{K}| = 0. \quad (3.6)$$

The zero roots of the characteristic equation are the eigenvalues of the system that determine the modal damping σ_j and frequency ω_j of the system modes. As the electric current is increased, the damping of each mode (σ_j) turns to be positive at a specific current value, and the mode becomes unstable. The first instability point determines the critical electric current and the instability mode. Therefore, if real parts of all eigenvalues are negative, the system is stable. Namely, the trivial solutions are asymptotically stable. If there exists at least one eigenvalue with a positive real part, then there are infinitely growing solutions for arbitrary small initial conditions and the trivial solution is unstable.

Since the considered system is conservative and there is no any external energy source in the system, the static instability or divergence occurs. Namely, the unstable mode grows as $e^{\sigma_j t}$, where σ_j is a positive real number. Thus the amplitude of the unstable mode (and consequently the GNR's deflections) will be increased exponentially as shown in Fig. 3. Note that in practice, the nonlinear effects inhibit the large deflections and thus the predicted instability is for the linearized system (the onset of instability). This type of instability is exactly similar to the buckling of the beams under a compressive force.

4 Numerical results and discussion

Based on the obtained formulation, the variation of resonance frequencies of the GNR versus the electric current and length-to-width ratio is investigated.

Fig. 4 shows the dimensionless frequency of the fundamental mode versus the electric current for several length-to-width ratios when $b = 75nm$. The bending stiffness for the

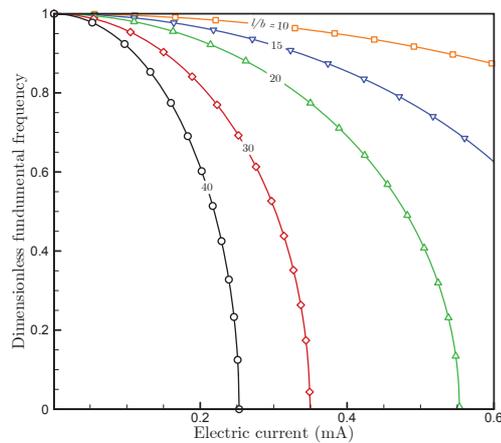


Figure 4: Dimensionless fundamental frequency versus the electric currents for several length-to-width ratios when $b = 75\text{nm}$.

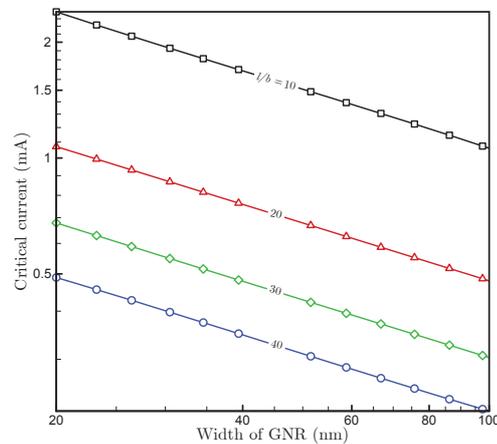


Figure 5: Critical electric current versus the width of the GNR for several length-to-width ratios in logarithmic scale.

GNR is taken as 2.1eV which is reported by Cranford and Buehler [16] based on the molecular dynamics results. The fundamental frequency is made dimensionless relative to its value at the zero current. The results reveal that the electric current and length-to-width ratio are crucial factors to determine the resonance frequency and the stability boundary of the GNR. Increasing the electric current yields the reduction of the resonance frequencies so that the buckling instability occurs at a specific current value. Eq. (3.4) confirms that the stiffness of the GNR decreases directly proportional to the square of the electric current. Furthermore, the results imply that increasing length-to-width ratio decreases the resonance frequency of the GNR.

Fig. 5 gives a better insight into the effect of width of the GNR as well as length-to-width ratio on the critical electric current in logarithmic scale. The areas below the curves

demonstrate the stable region that diminishes as length-to-width ratio and the width of the GNR decrease. Based on curve fitting to the presented results and several other case studies, the relation between the critical current (I_{cr}), the width and length of the GNR is obtained as described $I_{cr}(\text{mA}) = 155b^{0.666}/l^{1.166}$ in which the GNR's length and width are in nanometer.

5 Conclusions

In summary, the resonance frequency and the stability boundaries of a long graphene nano-ribbon carrying electric current were investigated. The results show self induced transverse Lorentz force on the GNR due to the interaction of the electric current with its own magnetic field causes the reduction of natural frequencies and increasing the current results in the divergence instability. Also, based on curve fitting to the obtained results an empirical relation between the critical current and the width and the length-to-width ratio of the GNR was introduced.

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