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A Stochastic Collocation Method for Delay Differential Equations with Random Input

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Abstract. In this work, we concern with the numerical approach for delay differential equations with random coefficients. We first show that the exact solution of the problem considered admits good regularity in the random space, provided that the given data satisfy some reasonable assumptions. A stochastic collocation method is proposed to approximate the solution in the random space, and we use the Legendre spectral collocation method to solve the resulting deterministic delay differential equations. Convergence property of the proposed method is analyzed. It is shown that the numerical method yields the familiar exponential order of convergence in both the random space and the time space. Numerical examples are given to illustrate the theoretical results.

AMS subject classifications: 65C20, 65M70

Key words: Delay differential equations, stochastic collocation, sparse grid, legendre spectral method.

1 Introduction

Using delay differential equations (DDEs) to model biological/engineer systems has a long history, dating to Malthus, Verhulst, Lotka and Volterra. Recently, DDE models have been arisen in many diverse applications including infectious disease dynamics including primary infection [11], immune response [24], tumor growth [23] and neural networks [4], to name a few. As the primary goal for using these models is to better our understanding of real word phenomena, it is becoming clear that the simple models can not capture the whole dynamics observed in natural systems. Thus, in real applications, the systems used are usually build up by a large number of DDEs with a lot of given data.

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The linear systems of DDEs admit the following form

$$\mathbf{u}'(t)=\mathbf{A}(t)u(t)+\mathbf{B}(t)\mathbf{u}(t-q(t)), \quad t\in I:=[0,T], \tag{1.1}$$

where $\mathbf{u}=(u_1,\cdots,u_N)^T$ is a vector of N unknown functions, and $\mathbf{A}$, $\mathbf{B}$ are coefficient matrices in $\mathbb{R}^{N\times N}$. The delay function $q(t)$ is assumed to satisfy

$$0<q(t)<t, \quad t\in(0,T], \quad q(0)=q\geq 0.$$

There are various types of delays in real applications [3], and here we will consider two widely used types of them, namely,

$$\text{Constant delay:} \qquad q(t)=\tau>0, \tag{1.2a}$$
$$\text{Pantograph delay:} \qquad t-q(t)=qt, \quad 0<q<1. \tag{1.2b}$$

The following initial conditions are needed for problem (1.1)

$$\mathbf{u}(t)=\mathbf{u}_0, \quad t\in[-\tau,0], \qquad \text{for constant delay}, \tag{1.3a}$$
$$\mathbf{u}(0)=\mathbf{u}_0, \qquad \text{for pantograph delay}. \tag{1.3b}$$

Generally speaking, the value of interest $\mathbf{u}$ is computed based on the input data $(\mathbf{A}(t),\mathbf{B}(t),\mathbf{u}_0)$ which are provided mainly by experimental measurements or a priori knowledge. It turns out that in many practical applications the input data are not known precisely a priori, which due to error in experiments and/or less of knowledge, namely, uncertainty in the given data. A popular way to deal with such an issue is to model these uncertain data as random variables/random functions [8]. For the context of delay differential equations, this will introduces the following random/parametric delay differential equations

$$\begin{cases} \mathbf{u}'(t,\vec{\xi})=\mathbf{A}(t,\vec{\xi})\mathbf{u}(t,\vec{\xi})+\mathbf{B}(t,\vec{\xi})\mathbf{u}(t-q(t),\vec{\xi}), \quad t\in I:=[0,T], \\ \mathbf{u}(t)=\mathbf{u}_0, \quad t\in[-q,0], \quad \text{for constant delay}, \\ \mathbf{u}(0)=\mathbf{u}_0, \quad \text{for pantograph delay}, \end{cases} \tag{1.4}$$

where $\vec{\xi}=(\xi_1,\cdots,\xi_M)$ is a random vector of M random parameters. One usually assumes that the random parameters are independent with each other, and further more, there is a corresponding probability density function $\rho(\xi_i)$ for each random parameter ξ_i in its supporting domain $[a_i,b_i]$.

Such a framework for problems with uncertain input has been widely used by researchers for partial/ordinary differential equations (PDE/ODE) models [8, 28, 29], and the resulting problems are also known as stochastic PDEs/ODEs. Stochastic modeling approaches for such problems can be categorized as either non-intrusive or intrusive. Intrusive approaches, such as generalized polynomial chaos methods (see, e.g., [8, 28] and references therein), usually result in deterministic coupled systems, and this require the