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Development of Lattice Boltzmann Flux Solver for Simulation of Incompressible Flows

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Abstract. A lattice Boltzmann flux solver (LBFS) is presented in this work for simulation of incompressible viscous and inviscid flows. The new solver is based on Chapman-Enskog expansion analysis, which is the bridge to link Navier-Stokes (N-S) equations and lattice Boltzmann equation (LBE). The macroscopic differential equations are discretized by the finite volume method, where the flux at the cell interface is evaluated by local reconstruction of lattice Boltzmann solution from macroscopic flow variables at cell centers. The new solver removes the drawbacks of conventional lattice Boltzmann method such as limitation to uniform mesh, tie-up of mesh spacing and time interval, limitation to viscous flows. LBFS is validated by its application to simulate the viscous decaying vortex flow, the driven cavity flow, the viscous flow past a circular cylinder, and the inviscid flow past a circular cylinder. The obtained numerical results compare very well with available data in the literature, which show that LBFS has the second order of accuracy in space, and can be well applied to viscous and inviscid flow problems with non-uniform mesh and curved boundary.

AMS subject classifications: 20B40

Key words: Chapman-Enskog analysis, flux solver, incompressible flow, Navier-Stokes equation, lattice Boltzmann equation.

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1 Introduction

Currently, for the simulation of incompressible viscous flows, most of numerical solvers can be roughly classified into two categories. One is based on the solution of Navier-Stokes (N-S) equations, while the other is based on the solution of lattice Boltzmann

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equation (LBE). N-S equations are from the application of mass and momentum conservation laws to a control volume. They have strong physical backgrounds. LBE is from the statistical physics. Both N-S solvers and LBE solvers have their distinguished features.

In the category of incompressible N-S solvers, the dependent variables are the macroscopic pressure and velocity. One approach in this category is the artificial compressibility method [1]. This method adds a weak compressibility into the incompressible N-S equations so that the well-established compressible N-S solvers can be applied to simulate incompressible flows. Its drawback is that the artificial compressibility involves a user-specified parameter, which may not be easy to give for some cases. The most popular solver in this category is to solve incompressible N-S equations directly. However, unlike compressible N-S equations, there is no transport equation for pressure in the incompressible N-S equations. In fact, the pressure is only appeared in the momentum equation but the velocity is involved in both the continuity and momentum equations. When the velocity is obtained from the momentum equation, there is no guarantee that it will satisfy the continuity equation. To overcome this difficulty, a number of algorithms [2–9], which are termed projection or pressure correction methods, have been proposed. These methods mainly resolve the coupling problem between the pressure field and the velocity field through the fractional step process. Usually, the process involves the solution of Poisson equation for pressure or pressure correction. The slow convergence of Poisson equation degrades the computational efficiency of this kind of N-S solvers, especially for unsteady flow simulation. In addition, to properly consider the effect of pressure oscillation in the numerical simulation, the staggered grid, on which the velocity components and pressure are defined at different locations, is often adopted. The use of staggered grid brings a great inconvenience in programming. Furthermore, as N-S equations are partial differential equations, N-S solvers need to use numerical schemes such as finite difference (FD), finite volume (FV) and finite element (FE) methods to discretize the first and second order spatial derivatives, and solve the resultant ordinary differential equations or algebraic equations. It is not a trivial job.

In contrast, LBE is a discrete model. At each physical location, a finite number of fictitious particles with given velocity (provided by lattice velocity model) are distributed. The density distribution functions of these particles are taken as unknowns, which can be determined from a set of algebraic equations (lattice Boltzmann equations). Once the density distribution functions are known at a physical location, the macroscopic flow variables such as density and velocity can be computed from mass and momentum conservation. LBE was initially developed by Chen et al. [10] and Qian et al. [11]. Since then, many variants of LBE have been developed in the literature [12–24]. Basically, LBE solver has two processes: streaming and collision. The streaming process involves particle distribution functions at two physical locations while the collision process happens locally. The collision process can be approximated by a linear model with a single relaxation time (BGK model) [10,11] or multi-relaxation times (MRT model) [16]. As compared with N-S solvers, the LBE solver has following distinguished features. Firstly, the linear streaming and collision processes of fictitious particles in the LBE solver can effectively consider the