Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **6**, No. 5, pp. 680-692

DOI: 10.4208/aamm.2014.5.s5
October 2014

Wave Interaction with an Emerged Porous Media

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Received 30 September 2013; Accepted (in revised version) 31 March 2014

Available online 23 July 2014

Abstract. In this paper, we study wave interaction with an emerged porous media. The governing equation is shallow water equations with a friction term of the linearized Dupuit-Forcheimer's formula. From the continuity of surface and horizontal flux, we derived the wave reflection and transmission coefficient formulas. They are similar with the corresponding formulas of the submerged solid bar breakwater. We solve the equations numerically using finite volume method on a staggered grid. The numerical wave reduction in the porous media confirms the analytical wave transmission curve.

AMS subject classifications: 76S05, 81U30, 35L02

Key words: Emerged porous media, shallow water equation, wave transmission coefficient, wave reflection coefficient.

1 Introduction

Porous structures, such as rubble-mound breakwaters are commonly used to protect harbors against the action of incident waves. Porous breakwaters have a large impact on waves and flow because they produce flow resistance. This resistance factor associated with the porous structure. For this purpose, before implementation in the real field, it is important to assess the engineering aspect of a porous structure, such as predicting the reflection and transmission wave coefficient correspond to the porous structure.

Studies about wave interaction with porous structure have been done by many authors, for instance R. A. Dalrymple et al. [3], M. Calabrese et al. [1], N. Kobayashi and Wurjanto [8], A. T. Chwang and A. T. Chan [2], W. Sulisz [19], P. J. Lynett et al. [11], P. L. F. Liu et al. [10], C. K. Sollitt and R. H. Cross [17]. The porous structures can be classified into two categories, submerged and emerged porous structure. Submerged breakwater lies

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entirely beneath the mean water level while emerged porous breakwater's crest is visible above the mean water level. N. Kobayashi and Wurjanto [8] studied monochromatic wave reflection and transmission over a submerged impermeable breakwater, Z. Gu and H. Wang [6,7] studied maximum wave energy dissipation by porous submerged breakwaters numerically using boundary integral element method. M. Calabrese et al. [1] has presented a method for calculating 2D wave setup behind a submerged breakwater. We also have studied wave interaction with a submerged porous structure in [16,21]. In this paper, we are interested to study wave interaction with an emerged porous structure.

Two common approaches to study wave reflection and transmission due to a porous structure are by using shallow water assumption with an additional friction and potential theory. W. Sulisz [19] predicted the reflection and transmission coefficient by using potential theory. The potential theory was also adopted in many literatures to investigate reflection and transmission from porous structure, such as O. S. Madsen [12] and R. A. Dalrymple et al. [3]. C. K. Sollitt and R. H. Cross [17] formulated the wave transmission through a permeable breakwater as a linear boundary value problem. A. T. Chwang and A. T. Chan [2] studied waves moving past a porous structure by using potential theory with Darcy's Law. P. L. F. Liu et al. [10] and P. J. Lynett et al. [11] studied solitary wave interaction with porous structure by using potential theory with Dupuit-Forcheimer friction.

In this paper, we study the problem of surface gravity waves reduce due to emerged porous media. We take the Shallow Water Equation (SWE) with an additional friction force of Dupuit-Forcheimer type. Next, we will formulate wave transmission and reflection coefficients. Based on the linear wave theory, the wave has a constant frequency and we can apply separation variable. In this way, we can directly get wave reduction from dispersion relation. When we combine solution in free water area and the reduced wave inside the porous structure, and from the continuity of surface and horizontal flux, we obtain explicit formulas for wave transmission and reflection coefficient. Numerically, we solve the equation using the finite volume method on a staggered grid. Numerical result of wave transmission coefficient is in good agreement with analytical data.

2 Model formulation

In this section, the governing equation of the flow pass through a porous structure will be formulated. Let η and u denote surface elevation and horizontal fluid velocity, respectively. We consider, the continuity and momentum equations in free region reads as:

$$\eta_t + (hu)_x = 0, \tag{2.1a}$$

$$u_t + g\eta_x = 0, \tag{2.1b}$$

with g is the gravitational acceleration. Notation $h = \eta + d$ denotes water thickness, where d is bottom topography.

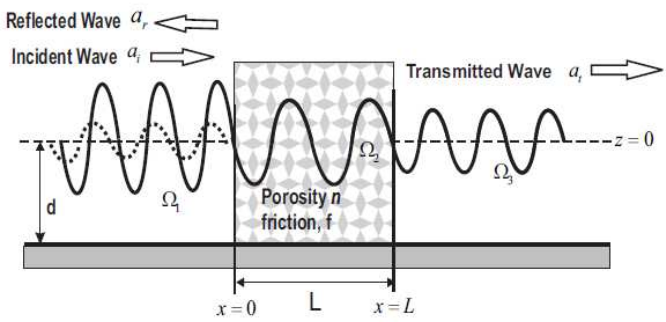


Figure 1: Sketch of the domain.

For flow in a porous media with porosity n, the rate of change of free surface η depends on the filtered horizontal momentum with filtered velocity u/n, where $0<n\le 1$. The momentum equation (2.1b) also gets an additional resistance by porous structure. Here, we implement a frictional force formulated by Dupuit-Forchheimer $(\alpha+\beta|u|)u$. Then, the full governing equations in the porous media are:

$$\eta_t+\Big(h\frac{u}{n}\Big)_x=0, \tag{2.2a}$$

$$\frac{1}{n}u_t+g\eta_x=-(\alpha+\beta|u|)u. \tag{2.2b}$$

The coefficient α expresses the laminar flow resistance whereas β expresses the turbulent flow resistance. Under the assumption that the waves are periodic, relatively long, and do not break, Madsen and White [12] approximate the non-linear friction term by linearizing it as:

$$u(\alpha+\beta|u|)\simeq f\frac{\omega}{n}u,$$

where f is the friction coefficient and ω is the wave frequency.

Further, we will consider wave elevation in a domain as depicted in Fig. 1. For that purpose, we denote a domain with porous media as Ω_2, whereas upstream and downstream regions are denoted Ω_1 and Ω_3, respectively. We recapitulate the governing equations for the whole domain are as follows:

$$\eta_t+\Big(h\frac{u}{N}\Big)_x=0, \tag{2.3a}$$

$$\frac{1}{N}u_t+g\eta_x+F_l\frac{\omega}{N}u=0, \tag{2.3b}$$

with piecewise constant functions of N and F_1 as follows

$$N=\begin{cases} 1, & \text{if } x\in\Omega_1 \text{ and } x\in\Omega_3, \\ n, & \text{if } x\in\Omega_2, \end{cases} \tag{2.4}$$

and

$$F_l = \begin{cases} 0, & \text{if } x \in \Omega_1 \text{ and } x \in \Omega_3, \\ f, & \text{if } x \in \Omega_2. \end{cases} \tag{2.5}$$

At the interfaces $x=0$ and $x=L$, the boundary condition comes from continuity of surface elevation η and horizontal flux hu.

3 Wave reflection and transmission

In this section, we will limit ourselves to the study of small wave amplitude in shallow water. We approximate $h=\eta+d$ in Eq. (2.2a) with d. Hence, Eqs. (2.2a) and (2.2b) with linear friction reduce to:

$$\eta_t + \left(d\frac{u}{n}\right)_x = 0, \tag{3.1a}$$

$$\frac{1}{n}u_t + g\eta_x + f\frac{\omega}{n}u = 0. \tag{3.1b}$$

Next, we consider a harmonic wave with certain frequency ω in porous domain read as:

$$\eta(x,t) = F(x)\exp(i\omega t), \tag{3.2a}$$

$$u(x,t) = G(x)\exp(i\omega t). \tag{3.2b}$$

Substituting Eqs. (3.2a), (3.2b) into Eqs. (3.1a), (3.1b) will yield:

$$F_{xx} + \frac{\omega^2}{gd}(1-if)F = 0, \tag{3.3a}$$

$$G(x) = -\frac{gn}{\omega}\frac{1}{i+f}F_x. \tag{3.3b}$$

Solution of Eq. (3.3a) is $F(x) = a_1 e^{-i\kappa x} + a_2 e^{i\kappa x}$, where the wave number in porous media κ follows from the following dispersion relation:

$$\kappa^2 = \frac{\omega^2}{gd}(1-if). \tag{3.4}$$

Solution κ of (3.4) has the form of $a-bi$ and $-a+bi$, where

$$a = \frac{w(1+f^2)^{1/4}}{gd}\cos\Big(\frac{\tan^{-1}(f)}{2}\Big) \quad \text{and} \quad b = \frac{w(1+f^2)^{1/4}}{gd}\sin\Big(\frac{\tan^{-1}(f)}{2}\Big).$$

A finite formulation for surface elevation is then:

$$\eta(x,t) = a_1 e^{-i(\kappa x - \omega t)} + cc. + a_2 e^{i(\kappa x + \omega t)} + cc.,$$

with $cc.$ denotes complex conjugate. Differentiate $F(x)$ with respect to x and substitute the result into (3.3b) will give us

$$G(x)=\frac{gn}{\omega}\frac{i\kappa}{i+f}\big(a_1e^{-i\kappa x}-a_2e^{i\kappa x}\big). \tag{3.5}$$

Next, substituting (3.5) into (3.2b) will yield:

$$u(x,t)=\sqrt{\frac{g}{d}}\frac{n}{\sqrt{1-if}}\big(a_1e^{-i(\kappa x-\omega t)}+cc.-a_2e^{i(\kappa x+\omega t)}+cc.\big). \tag{3.6}$$

Consider an incoming wave from free water area propagate to the right. After passing through the porous media the wave is partially reflected. So, in the upstream free region Ω_1 there will be right running wave $e^{-i(kx-\omega t)}$ and left running wave $e^{i(kx+\omega t)}$, where k denotes wave number in free region. The right running wave has amplitude a_i and left running wave has amplitude a_r. Then, in downstream free region Ω_3, after the wave has passed through the porous media, there is only right running transmitted wave, say with amplitude a_t. In general, surface elevation in the whole domain can be formulated as:

$$\eta(x,t)=e^{i\omega t}\begin{cases} a_ie^{-ikx}+a_re^{ikx} & \text{in } x<0,\\ a_1e^{-i\kappa x}+a_2e^{i\kappa x} & \text{in } 0\le x<L,\\ a_te^{-ikx} & \text{in } x>L,\end{cases} \tag{3.7}$$

with k follows dispersion relation $\omega^2/gk=kd$.

In free region, traveling waves of the linear SWE has to satisfy the following relation $u(x,t)=\pm\sqrt{g/d}\,\eta(x,t)$, with positive sign for right running wave and negative sign for left running wave. In porous domain the relation is

$$u(x,t)=\pm\sqrt{\frac{g}{d}}\frac{n}{\sqrt{1-if}}\eta(x,t).$$

Consider $\eta(x,t)$ formulated as (3.7), suitable anzat for $u(x,t)$ is as follows:

$$u(x,t)=e^{i\omega t}\begin{cases} \sqrt{\frac{g}{d}}(a_ie^{-ikx}-a_re^{ikx}) & \text{in } x<0,\\ \sqrt{\frac{g}{d}}\frac{n}{\sqrt{1-if}}(a_1e^{-i\kappa x}-a_2e^{i\kappa x}) & \text{in } 0\le x<L,\\ \sqrt{\frac{g}{d}}a_te^{-ikx} & \text{in } x>L.\end{cases} \tag{3.8}$$

Along the interface $x=0$ and $x=L$, free surface flow admits continuity of free surface η and horizontal momentum du.

Further, the coefficients a_1, a_2, a_r and a_t can be found by matching η and du at the two interfaces. Continuity of surface elevation η and horizontal flux du at $x=0$, will yield

$$a_1+a_2=a_i+a_r, \tag{3.9}$$

and

$$\epsilon(a_1-a_2)=a_i-a_r. \tag{3.10}$$

Similarly, continuity at $x=L$ will yield

$$a_1e^{-i\kappa L}+a_2e^{i\kappa L}=a_te^{-ikL}, \tag{3.11}$$

and

$$\epsilon(a_1e^{-i\kappa L}-a_2e^{i\kappa L})=a_te^{-ikL}, \tag{3.12}$$

with $\epsilon=n/\sqrt{1-if}$. From Eqs. (3.11) and (3.12), we can express a_1 and a_2 in terms of a_t:

$$a_1=\frac{1+\epsilon}{2\epsilon}e^{-ikL}e^{i\kappa L}a_t, \tag{3.13a}$$

$$a_2=\frac{\epsilon-1}{2\epsilon}e^{-ikL}e^{-i\kappa L}a_t. \tag{3.13b}$$

Substituting the results above for a_1 and a_2 into Eqs. (3.9) and (3.10), will give us two equations in three variables a_i, a_r, and a_t. Eliminating a_r and a_t will give us wave reflection and transmission coefficients, respectively:

$$K_R=\left|\frac{a_r}{a_i}\right|=\left|\frac{S^2-D^2}{S^2MR-D^2MR^{-1}}\right|, \tag{3.14a}$$

$$K_T=\left|\frac{a_t}{a_i}\right|=\left|\frac{SD(R-R^{-1})}{S^2R-D^2R^{-1}}\right|, \tag{3.14b}$$

where $S=1+\epsilon$, $D=1-\epsilon$, $M=e^{-ikL}$, and $R=e^{i\kappa L}$. Transmission and reflection coefficient are both depend on the characteristic of porous media such as porosity n, friction f, and length L of the porous media. Formulas (3.14a) dan (3.14b) are similar with wave reflection and transmission formulas for the case of submerged solid bar, derived by C. C. Mei [13], see also [5,14,22]. In the submerged solid bar case, the incident wave scattered because of depth discontinuity. Here, the incident wave scattered because it enters an emerged porous media.

Formulas (3.14a) and (3.14b) are tested for the following two limiting cases. Case $n\to 0$, the porous media becomes a solid wall and we obtain $K_R\to 1$ and $K_T\to 0$ which mean perfect reflection and no transmission. Case $n\to 1$ and $f\to 0$, in which the porous media becomes a free region, we obtain $\epsilon\to 1$, then $K_R\to 0$ and $K_T\to 1$. This means no reflection and perfect transmission. Further, the dispersion relation (3.4) reduces to the well-known dispersion relation for gravity wave: $\omega^2/gk=kd$, as we expect.

Taking parameter values $\omega=3\pi$, $d=5$, $g=9.81$, $n=0.8$, $f=0.18$, dispersion relation (3.4) will give us a complex value wave number $\kappa=1.350421351-0.1205690968I$. A

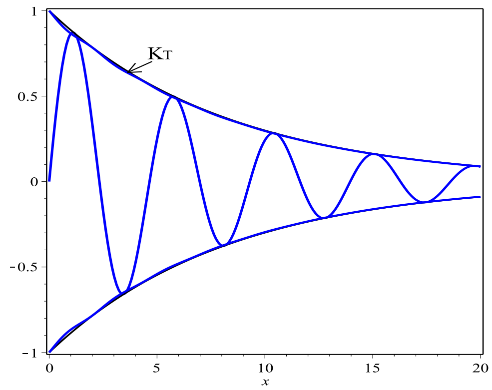


Figure 2: Solid line is the curve of $\eta(x,t)=\exp^{-i(\kappa x-\omega t)}$ at certain time for $\kappa=1.350421351-.1205690968I$ and its envelope $|\eta(x,t)|$ which is exactly K_T.

monochromatic wave $\exp^{-i(\kappa x-\omega t)}$ with negative imaginary part $\Im(\kappa)$ will undergo amplitude reduction, see Fig. 2. Let

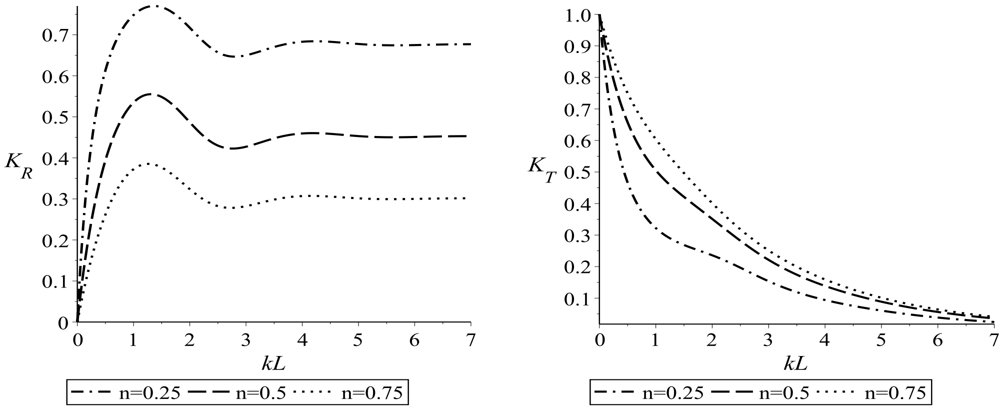
$$K_T=|\eta(x,t)|=\exp\Im(\kappa)x,$$

the term K_T denotes amplitude reduction of incident wave as a function of x, the horizontal length of an emerged porous breakwater. It also denotes the ratio between wave transmission amplitude and incident wave amplitude or wave transmission coefficient. It is clear that the profile of wave transmission coefficient depends strongly on the complex wave number κ, and hence on the dispersion relation (3.4). Parameters involve in (3.4) are wave frequency ω, gravitational acceleration g, porosity n, friction coefficient f, and water depth d.

Next, we study wave reflection and transmission coefficient with respect to porous structure parameter such as, porosity n, wave length L, and friction coefficient f. Here, we analyze the dependence of K_R and K_T on those parameters. Fig. 3 shows curves of K_R and K_T with respect to non-dimensional variable kL for several values of n. For all computations follow we take $g=9.8$ and $f=1$. We observe that the longer porous media does not directly mean larger K_R. For relatively small value of kL, we observe an oscillating behavior. The same behavior of K_R curve is also found by W. Sulisz [19]. We can also conclude that smaller n will yield smaller K_T.

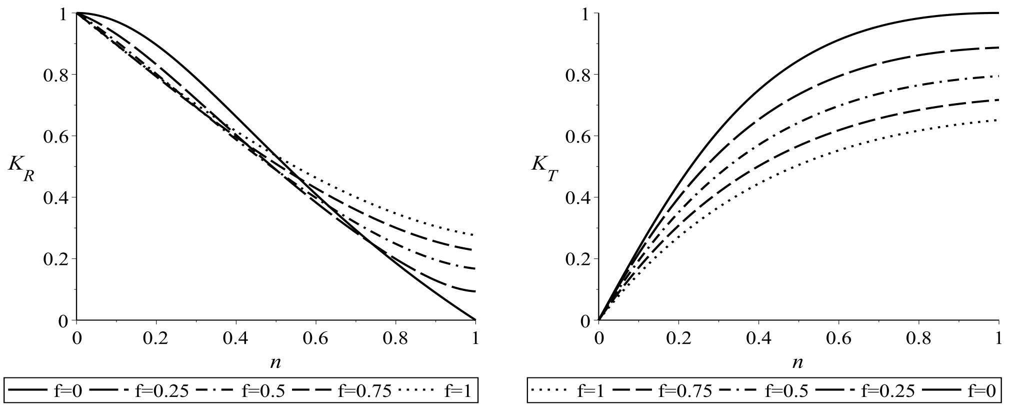
Fig. 4 shows curves of K_R and K_T with respect to n for several values of f. We conclude that larger f, will yield smaller K_T. And larger porosity n will yield larger K_T. We also conclude that smaller f will yield larger K_T. For large value of n, smaller f will yield smaller K_R.

Clearly, the wave reflection and transmission coefficient depend strongly on the porous structure parameters such as porosity n, length of the porous structure L, and friction coefficient f.



(a) (b)

Figure 3: (a) Curves of K_R w.r.t kL for fixed values of the friction coefficient $f=1$. (b) Curves of K_T w.r.t. kL for for fixed values of the friction coefficient $f=1$.



(a) (b)

Figure 4: (a) Curves of K_R w.r.t n for fixed values of $kL=1$. (b) Curves of K_T w.r.t. n for fixed values of $kL=1$.

4 A staggered finite volume method

In this section a numerical finite volume method on a staggered grid will be implemented for simulating an incident wave passing through an emerged porous media. We will use the numerical computations to confirm the analytical results.

Consider equation for gravity waves in a porous media (3.1a), (3.1b) in domain $[0,L]$. We discretize the porous domain in a staggered way $0=x_{1/2},x_1,\cdots,x_{Nx+1/2}=L$. Mass conservation (3.1a) is approximated at a cell centered at x_i whereas momentum conservation (3.1b) is approximated at a cell centered at $x_{i+1/2}$, see Fig. 5. Approximate equations are

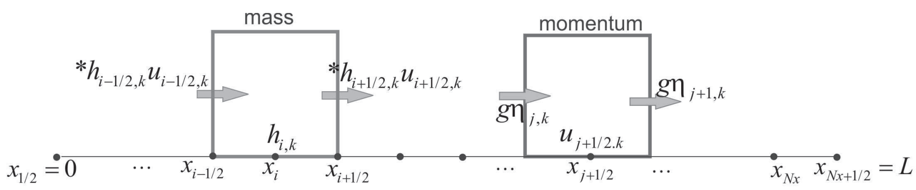


Figure 5: Illustration of staggered grid with cell $[x_{i-\frac{1}{2}},x_{i+\frac{1}{2}}]$ for mass conservation and cell $[x_{i-1},x_i]$ for momentum equation.

then

$$\frac{\eta_i^{n+1}-\eta_i^n}{\Delta t}+\frac{d}{n}\frac{u|_{i+1/2}^n-u|_{i-1/2}^n}{\Delta x}=0, \tag{4.1a}$$

$$\frac{\frac{1}{n}u_{i+1/2}^{n+1}-\frac{1}{n}u_{i+1/2}^n}{\Delta t}+g\frac{\eta_{i+1}^{n+1}-\eta_i^{n+1}}{\Delta x}+f\frac{\omega}{n}u_{i+1/2}^{n+1}=0. \tag{4.1b}$$

The above approach is known as the finite volume method on a staggered grid. This discretization is described extensively in G. S. Stelling and S. P. A. Duinmeijer [18]. In this setting, values of η will be computed at every full grid points x_i, with $i=1,2,\cdots,Nx$, using mass conservation (4.1a). Velocity u will be computed at every staggered grid points $x_{i+1/2}$, with $i=1,2,\cdots,Nx$, using momentum equation (4.1b). Implementing Von Neumann stability analysis, we obtain stability condition for (4.1a), (4.1b), which is $\sqrt{gd}\Delta t/\Delta x\le 1$, where d is the flat bottom depth. Note the friction term $f\omega u/n$ is calculated implicitly in order to avoid more restricted stability condition.

Further, for simulating the gravity waves in free water area, the approximate equations are just (4.1a) and (4.1b) with $n=1$ and $f=0$. The resulting scheme is free from numerical damping error, see S. R. Pudjaprasetya and I. Magdalena [15] for details.

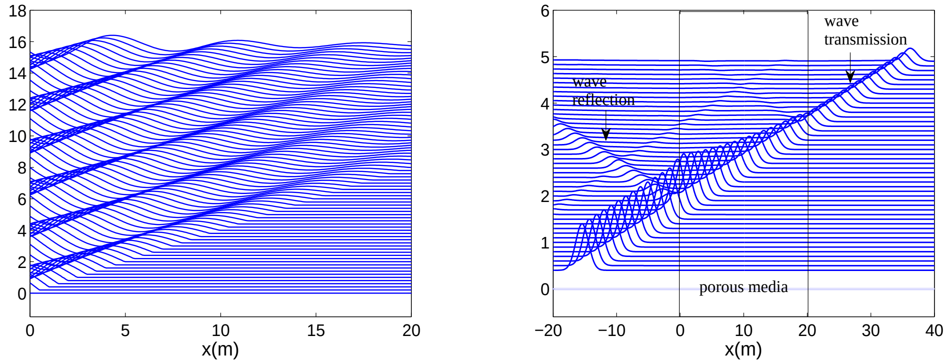
5 Numerical simulation

In this section, we will implement the above scheme to simulate wave interaction with a porous structure. For simulation, we take a computational domain $0<x<20$. We take $g=9.81$ and a constant depth $d=10$. The initial condition is still water level $\eta(x,0)=0$, $u(x,0)=0$ and for the left wave influx we take a monochromatic wave with amplitude 0.5

$$\eta(0,t)=0.5\sin 3\pi t. \tag{5.1}$$

Along the right boundary, we apply an absorbing boundary.

We first test the no porous case $n=1$ and without friction $f=0$, for which Eqs. (2.2a) and (2.2b) reduce to the shallow water equations without porous structure. Numerical simulation of (4.1a) and (4.1b) will yield a monochromatic wave travels undisturbed in shape, as we expect.



(a) (b)

Figure 6: (a) Damping of wave amplitude inside the porous media. (b) Wave interactions with an emerged porous media.

When the whole domain $0 \le x \le 20$ is a porous media with parameters $n = 0.8$, $f = 0.18$, and for computations we use $\Delta x = 0.1$, $\Delta t = \Delta x / \sqrt{gd} = 0.01$, then the results are given in Fig. 6(a). It shows wave amplitude reduction in the porous media.

Next, we will simulate the evolution of a wave initially being in a free water region. It travels to the right and enters a porous region. In the porous region the wave travels further to the right, and out to the free water region. For that purpose, we take a computational domain $-20 \le x \le 40$. Along the left and right boundaries, we apply absorbing boundary. The initial condition is the following hump:

$$\eta(x,0) = 1e^{-(0.5x-15)^2}, \tag{5.2a}$$

$$u(x,0) = \sqrt{\frac{g}{d}}\eta(x,0). \tag{5.2b}$$

The porous structure with parameters $n = 0.8$, $f = 0.18$ is installed in $0 < x < 20$. The numerical simulation is given in Fig. 6(b). When the waves hit the emerged porous media, it will split into reflected and transmitted waves. In the porous media, transmitted wave is reduced. When the wave travels further to the right and out to free water, it splits again into reflected and transmitted waves. This second reflected wave is very small and hardly noticeable in Fig. 6(b).

5.1 Comparison with analytical solution

In this section, we will show that our numerical surface profile reduces in the porous media with an envelope that confirms the analytical K_T curve. For numerical computations, we take the parameters used in Section 3, and we use $\Delta x = 0.1$, and $\Delta t = \Delta x / \sqrt{gd}$. The surface profile in a porous media is plotted in Fig. 7. Clearly, the numerical wave amplitude reduction confirms the analytical curve of K_T.

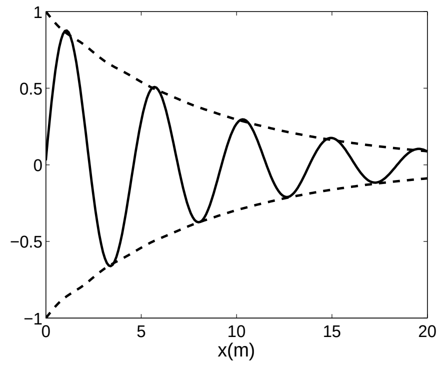


Figure 7: Solid line is the numerical surface elevation in porous media. Dash curves are the analytical wave transmission coefficient K_T from Eq. (3.14b).

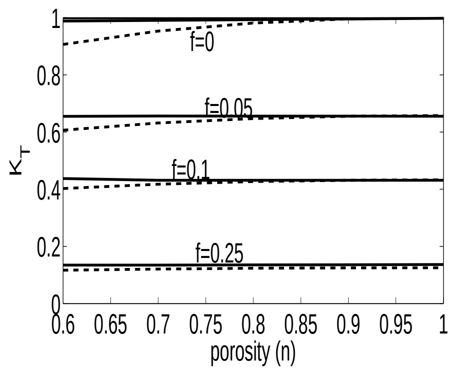


Figure 8: Comparison between wave transmission coefficient from numerics and analytic as function of porosity n for several friction coefficient f. Dash lines: analytical result, solid lines: numerical result.

Wave reflection and transmission coefficient, (3.14a) and (3.14b) depend on porosity n, friction coefficient f, length of the porous structure L, and wave number k, and κ. We do another comparison, the dependance of K_T with n and f. For the computation, we take parameter $\omega = 3\pi$ and $L = 10$. We plot the curve of wave transmission coefficient K_T with respect to porosity n for several values of f. From Fig. 8, we conclude that for a certain porosity n, larger friction coefficient, will lead to smaller wave transmission coefficient. The numerical results are in a good agreement with analytical result, especially for n larger than 0.8.

6 Conclusions

We have derived dispersion relation that holds for gravity waves in an emerged porous media. This dispersion relation explain diffusive mechanism of the porous structure. Further, wave reflection and transmission coefficients were also obtained. The formulas are similar to wave reflection and transmission coefficients from the solid submerged breakwater. Finite volume method on a staggered grid applied to SWE with linear friction result in numerical wave damping that confirms the analytical wave transmission coefficient.

Acknowledgments

We acknowledge financial support from riset dan inovasi KK ITB 122.21/AL-J/DIPA/PN/SPK/2013 and partially support from Riset Disentralisasi 1063c/l1.C01.2/PL/2014.

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