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On the Generalized Thermoelasticity Problem for an Infinite Fibre-Reinforced Thick Plate under Initial Stress

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Abstract. In this paper, the generalized thermoelasticity problem for an infinite fiber-reinforced transversely-isotropic thick plate subjected to initial stress is solved. The lower surface of the plate rests on a rigid foundation and temperature while the upper surface is thermally insulated with prescribed surface loading. The normal mode analysis is used to obtain the analytical expressions for the displacements, stresses and temperature distributions. The problem has been solved analytically using the generalized thermoelasticity theory of dual-phase-lags. Effect of phase-lags, reinforcement and initial stress on the field quantities is shown graphically. The results due to the coupled thermoelasticity theory, Lord and Shulman's theory, and Green and Naghdi's theory have been derived as limiting cases. The graphs illustrated that the initial stress, the reinforcement and phase-lags have great effects on the distributions of the field quantities.

AMS subject classifications: 73B, 73C, 73K

Key words: Dual-phase-lag theory, fiber-reinforced, initial stress, normal mode analysis, thick plate.

1 Introduction

In modern times, attention has been given to the problems of generation and propagation of elastic waves in an anisotropic elastic solids or layers of different configurations.

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There is a different between the isotropic and anisotropic media for the propagation of elastic waves. The information obtained from such studies is important to seismologists and geophysicists to find the location of the earthquakes as well as their energy, mechanism etc. and thereby gives valuable insight into the global tectonics. Available information suggests that the layered media, crystals and some other materials such as fiber-reinforced materials, fluid saturated porous materials etc. exhibits anisotropy. Belfied et al. [1] gave the idea of introducing a continuous self-reinforcement at every point of an elastic solid. Different problems concerning the surface waves in a fibre-reinforced anisotropic elastic media have been discussed in the literature [2–7].

The inclusion of the temperature change yields what so called the classical theory of thermoelasticity (Nowacki [8, 9]). The next step is to present the theory of coupled thermoelasticity. This was done by Biot [10] to overcome the first shortcoming of the classical theory. The third step is to modify the coupled thermoelasticity theory and to introduce a generalized thermoelasticity theory with one thermal relaxation (Lord and Shulman [11]). An extension is made by Green and Lindsay [12] to introduce two thermal relaxations for the generalized thermoelasticity theory. The fourth step is made by Green and Naghdi [13] to formulate the generalized theory of thermoelasticity without energy dissipation. The important step is made by Tzou [14–16] when he proposed the dual-phase-lag (DPL) model. This model includes two phase-lags, one of them is the heat flux τ_q and the other is the temperature gradient τ_θ. Many investigators have applied the DPL heat transfer model for different structures [17–20].

The wave propagation in solids subjected to initial stresses has been investigated by many authors for various models [4, 21]. In this article, the dual-phase-lag (DPL) generalized thermoelasticity theory is applied to study the 2-D problem of a fiber-reinforced thick plate subjected to initial stress. The problem is solved numerically using a normal mode analysis method. Numerical results for the temperature, displacements, and stresses distributions are illustrated graphically. The results obtained for field quantities may be used as benchmarks for future comparisons. They offer a significant theoretical basis and suggestions for the design of various fiber-reinforced thermoelastic elements under load to meet special engineering needs.

2 Basic equations

The linear governing equations of homogeneous, transversely isotropic, fiber-reinforced solid are presented here. The solid subjected to hydrostatic initial stress and treated without the inclusion of incremental body forces and heat sources. The basic equations in the context of generalized thermoelasticity with dual-phase-lags take the following form.

The equations of motion are given by

$$\sigma_{ij,j}+\left(u_{i,k}\sigma^0_{kj}\right)_{,j}=\rho\frac{\partial^2 u_i}{\partial t^2}, \tag{2.1}$$