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A Fifth-Order Low-Dissipative Conservative Upwind Compact Scheme Using Centered Stencil

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Abstract. In this paper, a conservative fifth-order upwind compact scheme using centered stencil is introduced. This scheme uses asymmetric coefficients to achieve the upwind property since the stencil is symmetric. Theoretical analysis shows that the proposed scheme is low-dissipative and has a relatively large stability range. To maintain the convergence rate of the whole spatial discretization, a proper non-periodic boundary scheme is also proposed. A detailed analysis shows that the spatial discretization implemented with the boundary scheme proposed by Pirozzoli [J. Comput. Phys., 178 (2001), pp. 81–117] is approximately fourth-order. Furthermore, a hybrid methodology, coupling the compact scheme with WENO scheme, is adopted for problems with discontinuities. Numerical results demonstrate the effectiveness of the proposed scheme.

AMS subject classifications: 65M06, 76M20, 76Q05

Key words: High-order scheme, compact scheme, conservative scheme, low-dissipative scheme.

1 Introduction

In recent years, high-order compact schemes have attracted much attention in various fields, such as simulation of incompressible or compressible flows, computational aeroacoustics, and computational electromagnetics [1–4]. Compact schemes can be viewed as weighted averages of explicit difference schemes at several adjacent points. Hence, these schemes are implicit ones and often produce tri-diagonal or penta-diagonal linear systems which could be solved by many effective algorithms. Compared with those explicit finite difference schemes, compact schemes use narrower stencil and show better spectral resolution property when offering the same order of approximation. Therefore, for

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problems with wide range of spatial and temporal scales, compact schemes should be a good choice. For such multi-scale problems, spectral methods may do the best. However, they are used mostly for problems with simple geometries and boundary conditions. Another type of difference schemes, the dispersion-relation-preserving (DRP) schemes, optimizes the coefficients of the schemes in order to resolve short waves whose wavelength is small with respect to the computational grid [5]. Since for long waves even lower order scheme can do fairly well, DRP schemes can be another choice for the problems mentioned above. However, they perform poorly for longer distances of travel if optimization is too aggressive [6].

A systematic analysis to high-order accurate, central stencil schemes appropriate for problems with a wide range of scales was presented by Lele [7]. However, for advection dominated problems, central high-order compact schemes lack some robustness because there is no built-in dissipation term in the schemes. A filtering procedure could be used to cure this problem. Otherwise, an artificial viscosity term has to be added into the scheme to improve the robustness of the scheme. Another choice is using upwind schemes which have inherent dissipation. Since difference schemes usually achieve the highest convergence order on their grid stencils and no effort has been made to optimize the resolution properties, the stencils used by upwind compact schemes are often upwind biased [8, 9]. However, there are some researches about upwind schemes using centered stencils [10, 11], whose coefficients are no longer symmetric. Compared with the central schemes on the same stencils, these schemes are usually degraded by one convergence order, and a free parameter is left to optimize the resolution property.

For problems involving discontinuities, compact schemes mentioned above will produce oscillations whose amplitude does not decrease when the grid is refined. To overcome this difficulty, nonlinear procedures should be employed. One way is to modify the compact schemes by using the ideas of TVD, WENO [12], or other kinds of limiter [13]. Hybridization with shock capturing schemes could be another kind of ways which take into account high resolution, high efficiency and good robustness. There are some works about hybrid compact ENO/WENO schemes [11, 14, 15], among which the hybrid methodology of Ren et al. shows the best resolution [16, 17].

The compact schemes proposed by Lele [7] need special boundary treatment to maintain the global conservation. However, conservative compact schemes guarantee the conservation without additional treatment [14]. Furthermore, the shock capturing schemes are usually conservative which is necessary for converging to the right solution when the grid is refined. Therefore, it should be better to use a conservative scheme for hybridization with shock capturing schemes. But the order of boundary treatment of conservative schemes should be analyzed carefully because the two adjacent fluxes of the boundary point may employ different schemes, which will be shown in the present work.

In this paper, a set of conservative upwind compact schemes with a free parameter using a centered stencil are given firstly. Following the method in [10], this free parameter is set to satisfy the condition that the dissipation errors of the compact scheme are smaller or comparable to the corresponding phase errors. Next, the boundary treatment is given