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Note on the Stability of a Slowly Rotating Timoshenko Beam with Damping

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Abstract. This paper continues the senior author's previous investigation of the slowly rotating Timoshenko beam in a horizontal plane whose movement is controlled by the angular acceleration of the disk of the driving motor into which the beam is rigidly clamped. It was shown before that this system preserves the total energy. We consider the problem of stability of the system after introducing a particular type of damping. We show that the energy of only part of the system vanishes. We illustrate obtained solution with the critical case of the infinite value of the damping coefficient.

AMS subject classifications: 35L10, 35B35, 74K10

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1 Introduction

The stability of rotating beams has been the subject of several investigations during the last two decades. The majority of publications concentrated on Euler beam model, e.g., [1, 7, 8]. Various stability problems were in the scope of considerations in those papers. Adding damping operator to clamped–free Euler beam with shear force control model in [1] caused L^2–stability of the system. In paper [7] by J. Valverde and D. Garcia Vallejo additional effects of Coriolis forces are observed, and their influence on stability of the beam rotating with a critical angular velocity is investigated. In [8], N. Lesaffre, J. J. Sinou and F. Thouverez presented the stability analysis of a system composed of rotating beams on a flexible, circular fixed ring, using Routh-Hurwitz criterion.

Timoshenko beam model is a generalization of Euler beam model, taking into account additional rotation of a cross–section area. Again, many authors considered different stability aspects for this generalized object. In [9], M. Sabuncu and K. Evran considered

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rotating asymmetric cross–section Timoshenko beam, for which the effects of the shear coefficient, the beam length, coupling due to the center of flexure distance from the centroid and rotation on the stability are considered. In [10], S. S. Rao and R. S. Gupta investigated the rotating twisted and tapered Timoshenko beam and studied the effects on the stability of the system of twist, offset, speed of rotation and variation of depth and breadth taper ratios.

Since 1999, W. Krabs and G. Sklyar considered different controllability and stabilizability aspects of a special, undamped model of a rotating Timoshenko beam clamped to the motor disk in [3–6]. In [3], authors solved the problem of transferring the beam from a position of rest into another given position of rest within a given time. In [5], they showed how to choose a feedback control allowing to stabilize the system in a preassigned position of a rest. In [6], W. Krabs, G. Sklyar and J. Woźniak obtained conditions of exact controllability under the assumption that the physical parameter γ appearing in the model equation is rational.

In this paper, following the works [3–6], we consider the problem of stability of a slowly rotating Timoshenko beam with damping. We present different cases of a particular damping model. After a careful analysis, we show that in all considered cases, the resulting system is unstable.

The structure of the work is as follows. After Introduction, Section 2 reviews the fundamental definitions used by the transfer function method. Next, in Section 3, we introduce rotating Timoshenko beam model and analyze its stability after adding damping. In conclusions, the summary of the obtained results is given, together with the possible directions for further research.

2 Transfer function method

In this section we introduce fundamental definitions from [1,2] and explain transfer function method. We present them for self-containment of the work.

Laplace transform is an integral operator, which transforms a function $f(t)$ with a real argument $(t\geq 0)$ to a function $F(s)$ with complex argument s.

Definition 2.1. Let $f:[0,\infty)\to X$ (X-separable Hilbert space) have property that $e^{\beta t}f(t)\in L^1([0,\infty);X)$ for some real β. We call these Laplace-transformable functions and we define their Laplace transform F(s) by

$$F(s)=\int_0^\infty e^{-st}f(t)dt$$

for $s\in\overline{\mathbb{C}_\beta^+}:=\{s\in\mathbb{C}|\mathrm{Re}(s)\geq\beta\}$. Other notation of Laplace transform is $\mathcal{L}\{f(t)\}$ or $\hat{f}(s)$. In this paper we use notation $\hat{f}(s)$.