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Numerical Analysis of the Mixed 4th-Order Runge-Kutta Scheme of Conditional Nonlinear Optimal Perturbation Approach for the EI Niño-Southern Oscillation Model

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Abstract. In this paper, we proposes and analyzes the mixed 4th-order Runge-Kutta scheme of conditional nonlinear perturbation (CNOP) approach for the EI Niño-Southern Oscillation (ENSO) model. This method consists of solving the ENSO model by using a mixed 4th-order Runge-Kutta method. Convergence, the local and global truncation error of this mixed 4th-order Runge-Kutta method are proved. Furthermore, optimal control problem is developed and the gradient of the cost function is determined.

AMS subject classifications: 65M06, 65M12

Key words: The EI Niño-Southern Oscillation (ENSO) model, 4th-order Runge-Kutta scheme, optimal control problem, conditional nonlinear perturbation.

1 Introduction

Optimal control theory is a mature mathematical discipline with numerous applications in both science and engineering. With the development of society and progress of science, the development of efficient numerical methods for the optimal control theory is

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a fundamental component in the applied methematics. Moreover, it plays an important role in many current scientific, engineering, and industrial applications.

In this paper, we mainly consider theoretical model for EI Niño-Southern Oscillation (EISO) including prognostic equations for sea surface temperature and for themocline variation [1]. This model can reproduce essential aspects of ENSO evolution: cyclic, chaotic, and phase-locking with annual cycles. Moreover, two dimensionless equation is involved: one reveals the nonlinear evolution of the anomalous sea surface temperature in the equatorial western/estern Pacific, and the other show the variation of the anomalous thermocline depth. Here, a classical 4th-order Runge-Kutta method is applied and developed for the mixed coupled ocean-atmosphere model. Two 4th-order Runge Kutta scheme are respectively designed for two equations, which involve the nonlinear evolution, coupling with two physical objects. The convergence and global truncation model are analyzed and we here obtain the same results as the simple ODE for the ENSO model.

Many physical phenomena on the ocean-atmosphere can be viewed as perturbations on the basic flow for the scientific research. In mathematics, this can be viewed as development of initial perturbations' evolution. Especially, it attracts more experts on the stability, sensitivity and predictability studies in geophysical fluid dynamics. In the past several decades, linear singular vector (LSV) [2], the fastest growing perturbation of the linearized model, is one of the dominant tools with the assumption that the initial perturbation is sufficiently small. Thus, its evolution can be governed approximately by the tangent linear model (TLM) of a nonlinear model. However, LSV neglects important issues on the nonlinearity and complexity of the physical phenomena. Then, conditional nonlinear optimal perturbation (CNOP) [3,4] is introduced with the initial perturbation of nonlinear evolution. Recently, CNOP has be extend to a comprehensive approach for the optimal combined mode of initial perturbations and model parameter perturbations [5]. This method has been succesfully applied in weather forecast and climatic prediction including EI Niño-Southern Oscillation (EISO) [6,7].

Among these literatures related to CNOP, the conventional adjoint method is used to provide the gradient of cost function for the optimization process. Some iterative methods have been used and achieved the better performance. Here, we mainly choose the Broyden-Fletcher-Goldfarb-Shanno (L-BFGS) for the presented problem, which only uses a limited memory variation of the BFGS update to approximate the inverse Hessian matrix and thus has an advantage over other corresponding methods. Moreover, CMA-ES method is a derivative free optimizer, the derivation argument is omitted and there are some other named parameters to control the behaviour of the algorithm. We will discuss this algorithm for the discontinuous cost function in the further study. However, to the best of our knowledge, [3–7] are the papers where numerical experiments have been provided to be efficient, and no rigorous error analysis of these methods has been done yet. In this paper, we mainly analyze and develop the mixed 4th-order Runger-Kutta schme of CNOP approach for the ENSO model. The convergence and global truncation error are obtained. Finally, optimal control problem is developed and the gradient of the cost