

## Rock Stress Around Noncircular Tunnel: a New Simple Mathematical Method

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**Abstract.** A new simple mathematical method has been proposed to predict rock stress around a noncircular tunnel and the method is calibrated and validated with a numerical model. It can be found that the tunnel shapes and polar angles affect the applicable zone of the theoretical model significantly and the applicable zone of a rectangular tunnel was obtained using this method. The method can be used to predict the values of the concentrated stress, and to analyze the change rate of rock stress and back to calculate the mechanical boundary condition in the applicable zone. The results of the stress change rate indicate that the horizontal stress is negatively related to the vertical boundary load and positively related to the horizontal boundary load. The vertical stress is negatively related to the horizontal boundary load and positively related to the vertical boundary load. These findings can be used to explain the evolution of the vertical increment in stress obtained with field-based borehole stress monitoring.

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**Key words:** Rock stress, mathematical model, noncircular tunnel, applicable zone, stress change rate, numerical simulation.

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## 1 Introduction

In underground mining operations, openings are generally characterized as having rectangular or other noncircular tunnels. These geometries can result in difficulties and complexities in obtaining and using analytical solutions of rock stress [1, 2]. An analytical

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solution is often a better and more efficient way to analyze the influence of variables like the boundary conditions on the stress distribution around the tunnel compared with finite element or finite difference numerical methods and can be used to analyze monitored results accurately [3–5]. For example, some analytical solutions for circular tunnel have been obtained around anisotropic materials and widely used combined with some failure criteria [6–11].

The precious complex variable theory and conformal mapping method is widely used to solve the analytical solution of stress around a noncircular tunnel in underground engineering [12–15]. Using this method, Huo et al. [16] deduced an analytical solution and revealed the effect of the far-field shear stress on the stability of a rectangular structure around the rectangle tunnel. Combining with the power series method, the analytical solution of the stress around an arbitrary shaped tunnel section was obtained and the results agree well with that from the numerical simulation model [17]. By introducing a general form of mapping function and an arbitrary biaxial loading into the boundary conditions, the general analytical solution of the stress around the arbitrary shaped tunnel was successfully revealed and used [18].

A new simple method, referred to as “equivalent radius”, first put forward by Liu et al. [19] has become widely used in engineering analyses in China [20–22]. The applicability and the reasonability of this method for noncircular tunnels have not been adequately investigated. Therefore, using a rectangular tunnel, we assessed the new method for obtaining an equivalent radius for the theoretical model. We successfully solved the following problems: the establishment and verification of a theoretical model, the definition of an acceptable zone around the rectangle tunnel in which the new and original methods could be applied, and the application of a theoretical model for analyzing the stress change rate induced by the boundary load and in predicting the concentrated stress within the acceptable zone.

## 2 Method introduction

### 2.1 Theoretical model

Four elements: burial depth, lithology, boundary conditions and section shape, were considered to establish the theoretical model (Fig. 1). The sectional area of an underground opening is generally very small compared with the burial depth. In order to keep the model as simple as possible, the boundary conditions assumed pertaining to a uniformly distributed load ( $q_1$  and  $q_2$  in Fig. 1). Regardless of whether the rock being examined is igneous, sedimentary or metamorphic, it can be either isotropic or anisotropic. Here, we simply assume that the lithology around the openings is isotropic and homogeneous. Furthermore, we use the equivalent radius instead of a noncircular section shape like a rectangle.

The following values were established to calculate the stress around the noncircular openings: a circumcircle radius ( $r_c$ ), a constant representing the distance ( $r_d$ ) between