Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **9**, No. 6, pp. 1364-1382

DOI: 10.4208/aamm.OA-2016-0052
December 2017

An Adaptive Semi-Lagrangian Level-Set Method for Convection-Diffusion Equations on Evolving Interfaces

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Received 4 July 2016; Accepted (in revised version) 23 December 2016

Abstract. A new Semi-Lagrangian scheme is proposed to discretize the surface convection-diffusion equation. The other involved equations including the the level-set convection equation, the re-initialization equation and the extension equation are also solved by S-L schemes. The S-L method removes both the CFL condition and the stiffness caused by the surface Laplacian, allowing larger time step than the Eulerian method. The method is extended to the block-structured adaptive mesh. Numerical examples are given to demonstrate the efficiency of the S-L method.

AMS subject classifications: 65L50, 65M06, 65M20

Key words: Convection-diffusion equation, semi-Lagrangian method, level-set method, block-structured adaptive mesh, finite difference method.

1 Introduction

Solving PDEs on evolving interfaces has attracted a lot of attention in recent years (e.g., [6, 9, 11, 16, 17, 23, 26]). One of the important applications is the surfactant, which plays a critical role in numerous important industrial and biological applications (e.g., [28,30,32] and the references therein). In a fluid flow, surfactant is transported along the interface by advection via the tangential interface velocity as well by diffusion within the interface.

In [26], an Eulerian level-set method for convection-diffusion equations on evolving interfaces. The Eulerian method was applied in simulating interfacial/two-phase flows with insoluble surfactant in [28–31], two-phase flows with insoluble surfactant and moving contact line in [32], two-phase flows with insoluble surfactant under electric fields in [24], and the surface phase separation in an interfacial flow in [12]. However, all these

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works were done on uniform mesh. For practical problems, often some part of the computational domain needs higher resolution than the others. Uniform mesh refinement can be prohibitively expensive in both computer storage and computation time. Adaptive mesh refinement stragety has been an efficient way to overcome this difficulty by putting more grid points only in the under-resolved region.

It is well-known that the Eulerian level-set methods have the CFL stability constrain. On the other hand, The semi-Lagrangian (S-L) method of [5] is an alternative for problems with convection. In a S-L method, ordinary differential equations for the characteristic curves are solved backward to locate the departure points, then variable approximations at the grid points are obtained by appropriate interpolations. Thus the S-L method releases the CFL stability constraint, allowing larger time steps than its Eulerian counterpart. This feature together with the interpolation makes the S-L method convenient for adaptive mesh refinement.

In this work, we propose an adaptive finite difference level-set method for solving convection-diffusion equations on evolving surfaces. In particular, a new S-L scheme is proposed for solving the extended convection-diffusion equation. The level-set convection equation and the re-initialization are also solved by using S-L schemes. In the adaptive method, we use the block-structured adaptive mesh (BSAM) of [1–3]. The advantages of BSAM include relatively easy mesh generation and parallelization. BSAM is an important component of well-known software packages such as Chombo of [4], Bearclaw of [15] and Paramesh of [13].

The paper is organized as follows. The governing equations are given in Section 2. The S-L schemes on uniform mesh are presented in Section 3. The adaptive mesh method is described in Section 1. Numerical examples in 2D and 3D are shown Section 4. Conclusion is given in Section 5.

2 Problem description

The evolving surface $\Gamma(t)$ is represented by the zero level-set of a level set function $\phi(\mathbf{x},t)$. In this work, it is passively convected according to a given velocity $\mathbf{u}$. The level-set Hamilton-Jacobi (H-J) equation (see [18]) is following:

$$\frac{\partial \phi}{\partial t} + \mathbf{u}\cdot\nabla\phi = 0. \tag{2.1}$$

The dynamics of surfactant concentration f is governed by the convection-diffusion equation:

$$\dot{f} + f(\nabla_s \cdot \mathbf{u}) = D_s \nabla_s^2 f \quad \text{on } \Gamma, \tag{2.2}$$

where $\dot{f} = \frac{\partial f}{\partial t} + \nabla_s f \cdot \mathbf{u}$ is the material time derivative, $\nabla_s = (I - \mathbf{n}\otimes\mathbf{n})\nabla$ is the surface gradient, and D_s is the material diffusivity. The left-hand side of the equation corresponds to the convection of the material by the velocity field, which can be derived from the