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Novel Conformal Structure-Preserving Algorithms for Coupled Damped Nonlinear Schrödinger System

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Abstract. This paper introduces two novel conformal structure-preserving algorithms for solving the coupled damped nonlinear Schrödinger (CDNLS) system, which are based on the conformal multi-symplectic Hamiltonian formulation and its conformal conservation laws. The proposed algorithms can preserve corresponding conformal multi-symplectic conservation law and conformal momentum conservation law in any local time-space region, respectively. Moreover, it is further shown that the algorithms admit the conformal charge conservation law, and exactly preserve the dissipation rate of charge under appropriate boundary conditions. Numerical experiments are presented to demonstrate the conformal properties and effectiveness of the proposed algorithms during long-time numerical simulations and validate the analysis.

AMS subject classifications: 35Q55, 37K05, 37M15

Key words: Conformal conservation laws, conformal structure-preserving algorithms, coupled damped nonlinear Schrödinger system, dissipation rate of charge.

1 Introduction

A Bose-Einstein condensate (BEC) is a state of matter of bosons confined in an external potential and cooled to temperatures very near to absolute zero. Under such conditions, a large fraction of the atoms collapse into the lowest quantum state of the external potential, at which point quantum effects become apparent on a macroscopic scale. It was first predicted in 1924. Attention has recently broadened to include exploration of quantized vortex states and their dynamics associated with superfluidity [1, 2], and of systems of two or more condensates [3]. By a mean-field approximation, the state of

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the BEC can be described by the wave function of the condensate to dilute systems. At temperatures much smaller than the critical temperature for a two-component BEC, its wave function can be well described by two coupled nonlinear Schrödinger (CNLS) system [4–7]. The CNLS system also models beam propagation inside crystals or photo refractive as well as water wave interactions. Solitary waves in this system are often called vector solutions in the literature as they generally contain two components. It has been shown that, in addition to passing-through collision, vector solutions can also bounce off each other or trap each other [8]. As for numerical methods focusing on this type of problem, there have been a great deal of methods to solve it [9, 10]. Symplectic and multi-symplectic methods, which can preserve the geometric structures of the original problem under appropriate discretizations, have been paid attentions in recent decades [11–16]. In [17], the authors study the multi-symplectic Preissman scheme for C-NLS system. Aydın and Karasözen [18], consider the integration of CNLS equation with soliton solutions by a multi-symplectic six-point scheme. The CNLS system can be split into a linear multi-symplectic subsystem and a nonlinear Hamiltonian subsystem, then authors [19–21] discuss the multi-symplectic splitting methods for the problem. Besides the multi-symplectic conservation law, multi-symplectic Hamiltonian partial differential equations (PDEs) also have the energy and momentum conservation laws which play a crucial role in conservative PDEs. Wang et al. [22] propose the concept of local structure-preserving algorithms for PDEs, which are the natural generalization of the corresponding global structure-preserving algorithms. Then, Cai et al. [23, 24] and Gong et al. [25] generalize the idea of local structure-preserving algorithms and propose a lot of energy-preserving algorithms and momentum-preserving algorithms to solve multi-symplectic PDEs. Later, Li and Wu [26] investigate a general approach to constructing local energy-preserving algorithms which can be of arbitrarily high order in time for solving Hamiltonian PDEs, including the CNLS system.

We consider the two coupled damped nonlinear Schrödinger (CDNLS) system of the form

$$\begin{cases} i(\phi_t+\delta\phi_x)+\alpha\phi_{xx}+(|\phi|^2+\gamma|\psi|^2)\phi+i\frac{a}{2}\phi=0, \\ i(\psi_t-\delta\psi_x)+\alpha\psi_{xx}+(\gamma|\phi|^2+|\psi|^2)\psi+i\frac{b}{2}\psi=0, \end{cases} \tag{1.1}$$

where ϕ and ψ are complex amplitudes or "envelopes" of two wave packets, i is the imaginary unit, $x\in[x_L,x_R]$ and t are the space and time variables, respectively. The parameter δ is the normalized strength of the linear birefringent, γ is the cross-phase modulation coefficient which describes the minimum approximation of the transmission of light wave, and $a,b\geq 0$ are damping coefficients. As this linearly damped system, McLachlan and Perlmutter develop a reduction conformal theory and show that conformal symplectic methods generally preserve the conformal symplectic structure [27]. Then McLachlan and Quispel [28] extend this theory and construct some numerical conformal methods that preserve conformal properties, such as symplecticity and volume-preservation. Subsequently, Moore generalizes these results to multi-symplectic PDEs, and proposes a