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# Stability of Viscoelastic Wave Equation with Structural $\delta$-Evolution in $\mathbb{R}^n$

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**Abstract.** The aim of this paper is to study the Cauchy problem for the viscoelastic wave equation for structural $\delta$-evolution models. By using the energy method in the Fourier spaces, we obtain the decay estimates of the solution to considered problem.

**Key Words**: Viscoelatic wave equation, Fourier transform, Lyapunov functions, Decay rates.
**AMS Subject Classifications**: 35L05, 35B35

## 1 Introduction

Cavalcanti et al. [3] studied the equation

$$|u_t|^\rho u_{tt} - \Delta u - \Delta u_{tt} - \int_0^t g(t-s)\Delta u(s)ds - \gamma \Delta u_t = 0, \tag{1.1}$$

with $x \in \Omega$, $t > 0$, $\rho > 0$. They proved a global existence result for $\gamma \geq 0$, and an exponential decay for $\gamma > 0$. This last result has been extended to a situation, where a source term is competing with the strong damping mechanism and the one induced by the viscosity. For more details see [9]. The authors combined well known methods with perturbation techniques to show that a solution with positive small energy exists globally and decay to the rest state exponentially.

In any spaces dimension, the paper [13] treated the viscoelastic wave equation

$$u_{tt} - \Delta u + \int_0^t g(t-s)\Delta u(s)ds = 0, \quad x \in \mathbb{R}^n, \quad t > 0, \tag{1.2}$$

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where $g$, being positive and nonincreasing, is the relaxation function which describes the material in consideration and $u_0 = u(x,0)$ and $u_1 = u_t(x,0)$ are given data. By using the energy method in the Fourier spaces, the general decay estimates of the solution is shown. In [17], the author considered the following equation

$$\rho(x)\left(|u'|^{q-2}u'\right)' - M(\|\nabla_x u\|_2^2)\Delta_x u + \int_0^t g(t-s)\Delta_x u(s)ds = 0, \quad x\in\mathbb{R}^n, \quad t>0, \tag{1.3}$$

where $q,n \geq 2$ and $M$ is a positive $C^1$-function satisfying for

$$s\geq 0, \quad m_0>0, \quad m_1\geq 0, \quad \gamma\geq 1, \quad M(s)=m_0+m_1 s^{\gamma}.$$

In order to compensate the lack of Poincaré's inequality in $\mathbb{R}^n$ and for wider class of relaxation functions, the author used the weighted spaces to establish a very general decay rate of solutions of viscoelastic wave equations in Kirchhoff-type.

In [5], the author looked into a linear Cauchy viscoelastic equation with density. His study included the exponential and polynomial rates, where he used the spaces weighted by density to compensate for the lack of Poincaré's inequality. The same problem treated in [5], was considered in [7], where it id considered a Cauchy problem for a viscoelastic wave equation. Under suitable conditions on the initial data and the relaxation function, they prove a polynomial decay result of solutions. The used conditions on the relaxation function $g$ and its derivative $g'$ are different from the usual conditions.

Recently, in [8], the authors considered the weak-viscoelastic case in the following problem,

$$u'' - \Delta u - \Delta u' + \alpha(t)\int_0^t g(t-s)\Delta u(s,x)ds = 0, \quad x\in\mathbb{R}^n, \quad t\in\mathbb{R}_*^+, \tag{1.4}$$

where $n \geq 2$. The energy decay results were established for weak-viscoelastic wave equation in $\mathbb{R}^n$, which depends on the behavior of both $\alpha$ and $g$. The main idea of the proof was to construct an appropriate Lyapunov function of the system obtained after taking the Fourier transform.

To extend previous results, we study the decay rate of the solution to the Cauchy problem for structural damped $\delta$-evolution with memory term in Fourier spaces

$$u_{tt} + (-\Delta)^{\delta}u - \int_0^t g(t-s)(-\Delta)^{\delta}u(s)ds = 0, \quad x\in\mathbb{R}^n, \quad t>0, \tag{1.5}$$

with the initial conditions

$$u(x,0)=u_0(x), \quad u_t(x,0)=u_1(x), \quad \text{and} \quad \delta>1. \tag{1.6}$$

The model here considered are well known ones and refer to materials with memory as they are termed in the wide literature which is concerned about their physical, mechanical behavior and the many interesting analytical problems. The physical characteristic property of such materials is that their behavior depends on time not only through the present time but also through their past history.