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Global Existence of Large Data Weak Solutions for a Simplified Compressible Oldroyd–B Model Without Stress Diffusion

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Dedicated to Professor Weiyi Su on the occasion of her 80th birthday

Abstract. We start with the compressible Oldroyd–B model derived in [2] (J. W. Barrett, Y. Lu, and E. Süli, Existence of large-data finite-energy global weak solutions to a compressible Oldroyd–B model, Commun. Math. Sci., 15 (2017), 1265–1323), where the existence of global-in-time finite-energy weak solutions was shown in two dimensional setting with stress diffusion. In the paper, we investigate the case without stress diffusion. We first restrict ourselves to the corotational setting as in [28] (P. L. Lions, and N. Masmoudi, Global solutions for some Oldroyd models of non-Newtonian flows, Chin. Ann. Math., Ser. B, 21(2) (2000), 131–146) We further assume the extra stress tensor is a scalar matrix and we derive a simplified model which takes a similar form as the multi-component compressible Navier–Stokes equations, where, however, the pressure term related to the scalar extra stress tensor has the opposite sign. By employing the techniques developed in [30, 35], we can still prove the global-in-time existence of finite energy weak solutions in two or three dimensions, without the presence of stress diffusion.

Key Words: Compressible Oldroyd–B model, stress diffusion, weak solutions, negative pressure term.

AMS Subject Classifications: 76A05, 35D30, 35Q35, 76N10

1 Introduction

The Oldroyd–B model is a widely used constitutive model to describe the flow of viscoelastic fluids. Different variants of such models were studied from different points of

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view within many decades: in the past mostly for incompressible fluids, recently, however, also for the compressible ones. There are many results in the context of small data problems, on the other hand, the number of existence results for large data without any restriction on the size of the data or the length of the time interval are rather rare, even in the case of incompressible fluids. A typical feature for the viscoelastic fluid is the presence of an extra stress tensor which fulfils a certain type of transport equation. In most cases, the global-in-time existence results rely on the fact that an additional term describing the stress diffusion is present in these transport equations. Even though it is possible to justify the presence of the stress diffusion, in modelling, it is often neglected, as typically, such terms are many orders lower than other terms in the equations. Our aim in the present paper is to concentrate on a special case, where it is possible to neglect the stress diffusion in the compressible model and we can still obtain global-in-time existence of a solution without any restriction on the size of the data.

It is known that from the incompressible Navier–Stokes–Fokker–Planck system which is a micro-macro model describing incompressible dilute polymeric fluids one can derive the incompressible Oldroyd–B model in dumbbell Hookean setting, see [27]. A similar derivation can be performed in the compressible setting, see [2], where the existence of global-in-time finite-energy weak solutions was also shown in the two dimensional setting. However, an important role in the analysis of these models was played by the presence of the stress diffusion.

Let $\Omega \subset \mathbb{R}^d$ be a bounded open domain with a $C^{2,\beta}$ boundary (briefly, a $C^{2,\beta}$ domain), with $\beta \in (0,1]$, and $d = 2,3$. The compressible Oldroyd–B model derived in [2] posed in the time-space cylinder $Q_T := (0,T) \times \Omega$ is the following:

$$\partial_t \varrho + \mathrm{div}_x(\varrho \mathbf{u}) = 0, \tag{1.1a}$$

$$\begin{aligned}&\partial_t(\varrho \mathbf{u}) + \mathrm{div}_x(\varrho \mathbf{u} \otimes \mathbf{u}) + \nabla_x p(\varrho) - \mathrm{div}_x \mathbb{S}(\nabla_x \mathbf{u}) \\ &\quad = \mathrm{div}_x\big(\mathbb{T} - (kL\eta + \mathfrak{z}\eta^2)\mathbb{I}\big) + \varrho \mathbf{f},\end{aligned} \tag{1.1b}$$

$$\partial_t \eta + \mathrm{div}_x(\eta \mathbf{u}) = \varepsilon \Delta_x \eta, \tag{1.1c}$$

$$\partial_t \mathbb{T} + \mathrm{Div}_x(\mathbf{u}\mathbb{T}) - \Big(\nabla_x \mathbf{u} \mathbb{T} + \mathbb{T} \nabla_x^{\mathrm{T}} \mathbf{u}\Big) = \varepsilon \Delta_x \mathbb{T} + \frac{k}{2\lambda}\eta \mathbb{I} - \frac{1}{2\lambda}\mathbb{T}. \tag{1.1d}$$

Above, $\mathbb{S}(\nabla_x \mathbf{u})$ is the Newtonian stress tensor defined by

$$\mathbb{S}(\nabla_x \mathbf{u}) = \mu^S \left(\frac{\nabla_x \mathbf{u} + \nabla_x^{\mathrm{T}} \mathbf{u}}{2} - \frac{1}{d}(\mathrm{div}_x \mathbf{u})\mathbb{I} \right) + \mu^B (\mathrm{div}_x \mathbf{u})\mathbb{I}, \tag{1.2}$$

where $\mu^S > 0$ and $\mu^B \geq 0$ are the shear and bulk viscosity coefficients, respectively. The pressure p and the density ϱ of the solvent are supposed to be related by the typical power law relation:

$$p(\varrho) = a\varrho^\gamma, \quad a > 0, \quad \gamma \geq 1. \tag{1.3}$$