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$[\ell_p]_{e.r}$ Euler-Riesz Difference Sequence Spaces

Hacer Bilgin Ellidokuzoğlu[1,*] and Serkan Demiriz[2]

[1] *Department of Mathematics, Recep Tayyip Erdoğan University, Rize, Turkey*
[2] *Department of Mathematics, Tokat Gaziosmanpaşa University, Tokat, Turkey*

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Abstract. Başar and Braha [1], introduced the sequence spaces $\breve{\ell}_\infty$, $\breve{c}$ and $\breve{c}_0$ of Euler-Cesáro bounded, convergent and null difference sequences and studied their some properties. Then, in [2], we introduced the sequence spaces $[\ell_\infty]_{e.r}$, $[c]_{e.r}$ and $[c_0]_{e.r}$ of Euler-Riesz bounded, convergent and null difference sequences by using the composition of the Euler mean E_1 and Riesz mean R_q with backward difference operator Δ. The main purpose of this study is to introduce the sequence space $[\ell_p]_{e.r}$ of Euler-Riesz $p-$absolutely convergent series, where $1 \leq p < \infty$, difference sequences by using the composition of the Euler mean E_1 and Riesz mean R_q with backward difference operator Δ. Furthermore, the inclusion $\ell_p \subset [\ell_p]_{e.r}$ hold, the basis of the sequence space $[\ell_p]_{e.r}$ is constucted and $\alpha-$, $\beta-$ and $\gamma-$duals of the space are determined. Finally, the classes of matrix transformations from the $[\ell_p]_{e.r}$ Euler-Riesz difference sequence space to the spaces ℓ_∞, c and c_0 are characterized. We devote the final section of the paper to examine some geometric properties of the space $[\ell_p]_{e.r}$.

Key Words: Composition of summability methods, Riesz mean of order one, Euler mean of order one, backward difference operator, sequence space, BK space, Schauder basis, $\beta-$duals, matrix transformations.

AMS Subject Classifications: 40C05, 40A05, 46A45

1 Preliminaries, background and notation

By a sequence space, we understand a linear subspace of the space $w = \mathbb{C}^{\mathbb{N}}$ of all complex sequnces which contains ϕ, the set of all finitely non-zero sequences, where $\mathbb{N} = \{0, 1, \cdots\}$. We shall write ℓ_∞, c and c_0 for the spaces of all bounded, convergent and null sequences, respectively. Also by bs, cs, ℓ_1 and ℓ_p, we denote the spaces of all bounded, convergent, absolutely and $p-$absolutely convergent series, respectively, where $1 < p < \infty$.

*Corresponding author. *Email addresses:* hacer.bilgin@erdogan.edu.tr (H. Ellidokuzoğlu), serkandemiriz@gmail.com (S. Demiriz)

We shall assume throughout unless stated otherwise that $p,q>1$ with $p^{-1}+q^{-1}=1$ and $0<r<1$, and use the convention that any term with negative subscript is equal to naught.

Let λ,μ be two sequence spaces and $A=(a_{nk})$ be an infinite matrix of real or complex numbers a_{nk}, where $n,k\in\mathbb{N}$. Then, we say that A defines a matrix mapping from λ into μ, and we denote it by writing $A:\lambda\to\mu$, if for every sequence $x=(x_k)\in\lambda$ the sequence $Ax=\{(Ax)_n\}$, the $A-$transform of x, is in μ; where

$$(Ax)_n=\sum_k a_{nk}x_k, \quad (n\in\mathbb{N}). \tag{1.1}$$

By (λ,μ), we denote the class of all matrices A such that $A:\lambda\to\mu$. Thus, $A\in(\lambda,\mu)$ if and only if the series on the right hand side of (1.1) converges for each $n\in\mathbb{N}$ and every $x\in\lambda$, and we have $Ax=\{(Ax)_n\}_{n\in\mathbb{N}}\in\mu$ for all $x\in\lambda$. A sequence x is said to be $A-$summable to α if Ax converges to α which is called the $A-$limit of x.

Let X be a sequence space and A be an infinite matrix. The sequence space

$$X_A=\{x=(x_k)\in w: Ax\in X\} \tag{1.2}$$

is called the domain of A in X which is a sequence space.

A sequence space λ with a linear topology is called a $K-$ space provided each of the maps $p_i:\lambda\to\mathbb{C}$ defined by $p_i(x)=x_i$ is continuous for all $i\in\mathbb{N}$. A $K-$ space is called an $FK-$ space provided λ is a complete linear metric space. An $FK-$ space whose topology is normal is called a $BK-$ space. If a normed sequence space λ contains a sequence (b_n) with the property that for every $x\in\lambda$ there is a unique sequence of scalars (α_n) such that

$$\lim_{n\to\infty}||x-(\alpha_0 b_0+\alpha_1 b_1+\cdots+\alpha_n b_n)||=0,$$

then (b_n) is called a Schauder basis (or briefly basis) for λ. The series $\sum\alpha_k b_k$ which has the sum x is then called the expansion of x with respect to (b_n), and written as $x=\sum\alpha_k b_k$.

A matrix $A=(a_{nk})$ is called a triangle if $a_{nk}=0$ for $k>n$ and $a_{nn}\neq 0$ for all $n\in\mathbb{N}$. It is trivial that $A(Bx)=(AB)x$ holds for the triangle matrices A,B and a sequence x. Further, a triangle matrix U uniquely has an inverse $U^{-1}=V$, which is also a triangle matrix. Then, $x=U(Vx)=V(Ux)$ holds for all $x\in w$.

Let us give the definition of some triangle limitation matrices which are needed in the text. Δ denotes the backward difference matrix $\Delta=(\Delta_{nk})$ and $\Delta'=(\Delta'_{nk})$ denotes the transpose of the matrix Δ, the forward difference matrix, which are defined by

$$\Delta_{nk}=\begin{cases}(-1)^{n-k}, & n-1\le k\le n,\\ 0, & 0\le k<n-1 \quad \text{or} \quad k>n,\end{cases}$$

$$\Delta'_{nk}=\begin{cases}(-1)^{n-k}, & n\le k\le n+1,\\ 0, & 0\le k<n \quad \text{or} \quad k>n+1,\end{cases}$$

for all $k,n\in\mathbb{N}$; respectively.