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Boundedness on Triebel-Lizorkin and Lebesgue Spaces of Multilinear Singular Integral Operators Satisfying a Variant of Hörmander's Condition

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Abstract. The boundedness on Triebel-Lizorkin and Lebesgue spaces of the multilinear operators associated to some singular integral operators satisfying a variant of Hörmander's condition are obtained.

Key Words: Multilinear operator, singular integral operator, variant of Hörmander's condition, Triebel-Lizorkin space, Lipschitz space.

AMS Subject Classifications: 42B20, 42B25

1 Introduction

As the development of singular integral operators, their commutators and multilinear operators have been well studied (see, e.g., [2–6]). From [2, 9, 12], we know that the commutators and multilinear operators generated by the singular integral operators and the Lipschitz functions are bounded on the Triebel-Lizorkin and Lebesgue spaces. In [6], some singular integral operators satisfying a variant of Hörmander's condition are introduced, and the boundedness for the operators are obtained (see, e.g., [8, 16]). Motivated by these papers, in this paper, we will introduce some multilinear operators associated to certain singular integral operators satisfying a variant of Hörmander's condition and prove the boundedness properties for the multilinear operators on the Triebel-Lizorkin and Lebesgue spaces.

2 Notations and theorem

First, let us introduce some notations. Throughout this paper, Q will denote a cube of R^n with sides parallel to the axes. For a locally integrable function f, the sharp function of f

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is defined by

$$f^{\#}(x)=\sup_{Q\ni x}\frac{1}{|Q|}\int_Q|f(y)-f_Q|dy,$$

where, and in what follows,

$$f_Q=|Q|^{-1}\int_Q f(x)dx.$$

It is well-known that (see [7,14,15])

$$f^{\#}(x)=\sup_{Q\ni x}\inf_{c\in C}\frac{1}{|Q|}\int_Q|f(y)-c|dy.$$

For $1\le p<\infty$ and $0\le\eta<n$, let

$$M_{\eta,p}(f)(x)=\sup_{Q\in x}\left(\frac{1}{|Q|^{1-p\eta/n}}\int_Q|f(y)|^pdy\right)^{1/p}.$$

For $\beta>0$ and $p>1$, let $\dot{F}_p^{\beta,\infty}(R^n)$ and $\dot{\wedge}_\beta(R^n)$ be the homogeneous Triebel-Lizorkin and Lipschitz spaces (see [12]).

Definition 2.1. *Let $\Phi=\{\phi_1,\cdots,\phi_m\}$ be a finite family of bounded functions in R^n. For any locally integrable function f, the Φ sharp maximal function of f is defined by*

$$M_\Phi^{\#}(f)(x)=\sup_{Q\ni x}\inf_{\{c_1,\cdots,c_m\}}\frac{1}{|Q|}\int_Q\Big|f(y)-\sum_{j=1}^m c_j\phi_j(x_Q-y)\Big|dy,$$

where the infimum is taken over all m-tuples $\{c_1,\cdots,c_m\}$ of complex numbers and x_Q is the center of Q. For $\eta>0$, let

$$M_{\Phi,\eta}^{\#}(f)(x)=\sup_{Q\ni x}\inf_{\{c_1,\cdots,c_m\}}\left(\frac{1}{|Q|}\int_Q|f(y)-\sum_{j=1}^m c_j\phi_j(x_Q-y)|^\eta dy\right)^{1/\eta}.$$

Remark 2.1. We note that $M_\Phi^{\#}\approx f^{\#}$ if $m=1$ and $\phi_1=1$.

Definition 2.2. *Given a positive and locally integrable function f in R^n, we say that f satisfies the reverse Hölder's condition (write this as $f\in RH_\infty(R^n)$), if for any cube Q centered at the origin we have*

$$0<\sup_{x\in Q}f(x)\le C\frac{1}{|Q|}\int_Q f(y)dy.$$

In this paper, we will study a class of multilinear operators associated to some singular integral operators satisfying a variant of Hörmander's condition type integral operators as following (see [8,16]).