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(p,q)-Analogue of Mittag-Leffler Function with (p,q)-Laplace Transform

Alok Jain[1], Altaf Ahmad Bhat[2,*], Renu Jain[1] and Deepak Kumar Jain[3]

[1] *School of Mathematics and Allied Sciences, Jiwaji University, Gwalior (M.P.), India*
[2] *Department of Mathematical Sciences, Islamic University of Science and Technology, Awantipora Pulwama, J and K, India-192122*
[3] *Department of Engineering Mathematics and Computing, MITS, Gwalior (M.P.), India*

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Abstract. The aim of this paper is to define (p,q)-analogue of Mittag-Leffler Function, by using (p,q)-Gamma function. Some transformation formulae are also derived by using the (p,q)-derivative. The (p,q)-analogue for this function provides elegant generalization of q-analogue of Mittag-Leffler function in connection with q-calculus. Moreover, the (p,q)-Laplace Transform of the Mittag-Leffler function has been obtained. Some special cases have also been discussed.

Key Words: (p,q)-analogue of Mittag-Leffler function, (p,q)-Gamma function, q-calculus, (p,q)-derivative operator, (p,q)-Laplace transform.

AMS Subject Classifications: 33D05, 33D15, 33D60, 33C20

1 Introduction

Our translation of real world problems to mathematical expressions relies on calculus which has been generalized in several directions. A natural generalization of calculus, called fractional calculus was developed during eighteenth century which involved the differentiation and integration operations of arbitrary order, which is a sort of misnomer. In the beginning it did not develop sufficiently due to lack of applications. Over the years various applications of the concept were explored and the efforts were so rewarding that the subject itself has been categorized as a significant branch of applicable mathematics. It plays a significant role in number of fields such as physics, rheology, quantitative

*Corresponding author. *Email addresses:* alokjain1969@gmail.com (A. Jain), altaf.u786@gmail.com (A. A. Bhat), renujain3@rediffmail.com (R. Jain), ain_dkj@mitsgwalior.in (D. K. Jain)

biology, electro-chemistry, scattering theory, diffusion, transport theory, probability, elasticity, control theory, engineering mathematics and many others.

In order to stimulate more interest in the subject, many research workers engaged their focus on another dimension of calculus which sometimes called calculus without limits or popularly q-calculus. The q-calculus was initiated in twenties of the last century. Kac and Cheung's book [11] entitled "Quantum Calculus" provides the basics of such type of calculus. The fractional q-calculus is the q-extension of the ordinary calculus. The investigations of q-integrals and q-derivatives of arbitrary order have gained importance due to its various applications in the areas like ordinary fractional calculus, solutions of the q-difference (differential) and q-integral equations, q-transform analysis etc.

Hypergeometric functions evolved as natural unification of a host of functions discussed by analysts from the seventeenth century to the present day. Functions of this type may also be generalized using the concept of basic number. Over the last thirty years, a great resurgence of interest in q-functions has been witnessed in view of their application to number theory and other areas of mathematics and physics.

The Mittag-Leffler function is the generalization of Hypergeometric function which appear as solution of well-known fractional differential and integral equations representing some physical and physiological phenomena like diffusion, transport theory, probability, elasticity and control theory.

The (p,q)-shifted factorial is based on the concept of twin-basic number

$$[n]_{p,q} = \frac{(p^n - q^n)}{(p-q)}.$$

The basic number occurs in the theory of two parameter quantum algebras and has also been introduced independently in combinatorics. Several properties of this number were studied briefly in [12]. Around the same time as [12], Brodimas et al. [13] and Arik et al. [14] also independently introduced the (p,q)-number in the Physics literature, but in a very much less detailed manner. The (p,q)-identities thus derived, with doubling of the number of parameters, offer more choices for manipulations.

It is notable that many of the q-results can be generalized directly to (p,q)-results and once we have the (p,q)-results, the q-results can be obtained more easily by mere substitutions for the parameters instead of any limiting process as required in the usual q-theory.

We observe that the q-results are of course special cases of the (p,q)-results corresponding to choosing $p=1$. This also provides a new look for the q-identities.

The q-deformed algebra [15,16] and their generalization (p,q)-analogue [12,17] attract much attention of the research to increase the accessibility of different dimensions of (p,q)-analogue algebra. The main reason is that these topics stand for real life problems, in mathematics and physics, later to the theory of quantum calculus.

In the present paper, the authors attention is towards the (p,q)-analogue of Mittag-Leffler function with (p,q)-Laplace transform by using the generalization of the Gamma and Beta functions namely the (p,q)-Gamma and (p,q)-Beta functions.