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# INTEGRABILITY AND $L^1$-CONVERGENCE OF DOUBLE COSINE TRIGONOMETRIC SERIES

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**Abstract.** We study here $L^1$-convergence of new modified double cosine trigonometric sum and obtain a new necessary and sufficient condition for $L^1$-convergence of double cosine trigonometric series. Also, the results obtained by Moricz[1],[2] are particular cases of ours.

**Key words:** *$L^1$-convergence, conjugate Dirichlet kernel*

**AMS (2010) subject classification:** 42A20, 42A32

## 1 Introduction

We consider the double cosine series

$$f(x,y)=\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\lambda_j\lambda_k\ a_{jk}\cos jx\cos ky \tag{1.1}$$

on the positive quadrant $T^2=[0,\pi]\times[0,\pi]$ of the two dimensional torus, where $\lambda_0=\frac{1}{2}$ and $\lambda_j=1$ for $j=1,2,3,\cdots$ and $\{a_{jk}\}$ is a double sequence of real numbers.

We denote by

$$S_{mn}(x,y)=\sum_{j=0}^{m}\sum_{k=0}^{n}\lambda_j\lambda_k\ a_{jk}\cos jx\cos ky,\qquad m,n\geq 0$$

the rectangular partial sum of the series (1.1) and $f(x,y)=\lim\limits_{m+n\to\infty}S_{mn}(x,y)$.

We remind the reader the following classes of coefficient sequences due to [1].

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*Definition* 1.1[1]. We say that $\{a_{jk}\}$ belongs to the class $BV_2$ if

$$a_{jk} \to 0 \qquad \text{as} \qquad j+k \to \infty, \tag{1.2}$$

and

$$\sum_{j=0}^{\infty}\sum_{k=0}^{\infty} |\triangle_{11} a_{jk}| < \infty, \tag{1.3}$$

where

$$\triangle_{11} a_{j,k} = a_{j,k} - a_{j+1,k} - a_{j,k+1} + a_{j+1,k+1}.$$

The condition (1.2) implies that $\{a_{jk}\}$ is a null sequence while (1.3) implies that $\{a_{jk}\}$ is a sequence of bounded variation.

*Defintion* 1.2[1] A null sequence $\{a_{jk}\}$ belongs to the class $\mathcal{C}_2$ if for every $\varepsilon > 0$ there exists $\delta > 0$ such that for all $0 \le m \le M$ and $0 \le n \le N$, we have

$$C(m,M;n,N;\delta) := \int\int_{D_\delta} \left| \sum_{j=m}^{M}\sum_{k=n}^{N} D_j(x)D_k(y)\triangle_{11} a_{jk} \right| \mathrm{d}x\mathrm{d}y \le \varepsilon \tag{1.4}$$

or

$$\int\int_{D_\delta} \left| \sum_{j=m}^{\infty}\sum_{k=n}^{\infty} D_j(x)D_k(y)\triangle_{11} a_{jk} \right| \mathrm{d}x\mathrm{d}y \le \varepsilon, \qquad \forall\ m,n \ge 0,$$

where

$$D_\delta := T - (\delta,\pi] \times (\delta,\pi] = \{(x,y) : 0 \le x,y \le \pi \ \ \&\ \ \min(x,y) \le \delta\}.$$

*Definition* 1.3[1]. A double sequence $\{a_{jk}\}$ is said to be quasi-convex if

$$\sum_{j=0}^{\infty}\sum_{k=0}^{\infty} (j+1)(k+1)|\triangle_{22} a_{jk}| < \infty. \tag{1.5}$$

Moricz[1] introduced the following modified double cosine trigonometric sum

$$u_{mn}(x,y) = \sum_{j=0}^{m}\sum_{k=0}^{n} \lambda_j \lambda_k \Big(\sum_{i=j}^{m}\sum_{l=k}^{n} \triangle_{11} a_{il}\Big) \cos jx \cos ky \tag{1.6}$$

and studied the $L^1$-convergence of double cosine trigonometric series whose coefficients belong to the class $BV_2$, $\mathcal{C}_2$ and the class of quasi-convex coefficients by making use of $L^1$-convergence of these modified double cosine trigonometric sums.

We introduce here the following new modified rectangular partial sums $g_{mn}$ of the series (1.1)

$$g_{mn}(x,y) \quad = \quad \frac{a_{00}}{2} + \sum_{j=1}^{m}\sum_{k=1}^{n} \left\{ \sum_{r=j}^{m}\sum_{l=k}^{n} \triangle_{11} (a_{rl} \cos rx \cos ly) \right\}. \tag{1.7}$$