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APPROXIMATION PROPERTIES OF rth ORDER GENERALIZED BERNSTEIN POLYNOMIALS BASED ON $q-$CALCULUS

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Abstract. In this paper we introduce a generalization of Bernstein polynomials based on q calculus. With the help of Bohman-Korovkin type theorem, we obtain $A-$statistical approximation properties of these operators. Also, by using the Modulus of continuity and Lipschitz class, the statistical rate of convergence is established. We also gives the rate of $A-$statistical convergence by means of Peetre's type $K-$functional. At last, approximation properties of a rth order generalization of these operators is discussed.

Key words: *$q-$integers, $q-$Bernstein polynomials, $A-$statistical convergence, modulus of continuity, Lipschitz class, Peetre's type $K-$functional*

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1 Introduction

Phillips[7] in 1997 proposed $q-$Bernstein polynomials based on q calculus as

$$B_{n,q}(f;x) = \sum_{k=0}^{n} f\left(\frac{[k]}{[n]}\right) \begin{bmatrix} n \\ k \end{bmatrix} x^k (1-x)_q^{n-k-1}.$$

Very recently Heping[12] obtained Voronovaskaya type asymptotic formula for q-Bernstein operator. In 2002 Ostrovska S.[9], studied the convergence of generalized Bernstein Polynomials. Study of $A-$statistical approximation by positive linear operators is attempted by O.Duman, C.Orhan in [8].

First, we recall the concept of A-statistical convergence.

Let $A = (a_{jn})_{j,n}$ be a non-negative infinite summability matrix. For a sequence $x := (x_n)_n$, $A-$transform of the sequence x, denoted by $Ax := (Ax)_j$, is given by

$$(Ax)_j := \sum_{n=1}^{\infty} a_{jn}x_n,$$

provided that the series on the right hand side converges for each j. We say that A is regular (see [8]) if $\lim Ax = L$ whenever $\lim x = L$. Let A be a non-negative summability matrix. The sequence $x := (x_n)_n$ is said to be $A-$statistically convergent to a number L, if for any given $\varepsilon > 0$,

$$\lim_j \sum_{n:|x_n-L|\geq\varepsilon} a_{jn} = 0,$$

and we denote this limit by $st_A - \lim_n x_n = L$.

We also know that

1. (see [1],[4]) For $A := C_1$, the Cesàro matrix of order one defined as

$$c_{jn} := \begin{cases} \dfrac{1}{j}, & 1 \leq n \leq j, \\ 0, & n > j, \end{cases}$$

then A-statistical convergence coincides with statistical convergence.

2. Taking A as the identity matrix, A-statistical convergence coincides with ordinary convergence, i.e.

$$st_A - \lim_n x_n = \lim x_n = L.$$

2 Construction of Operator

Here we introduce a general family of $q-$Bernstein polynomials and compute the rate of convergence with help of modulus of continuity and Lipschitz class. Before introducing the operators, we mention certain definitions based on q-integers, for the DETAILS, see [10] and [11]. For each nonnegative integer k, the $q-$integer $[k]$ and the $q-$factorial $[k]!$ are respectively defined by

$$[k] := \begin{cases} (1-q^k)/(1-q), & q \neq 1, \\ k, & q = 1 \end{cases} .$$

and

$$[k]! := \begin{cases} [k]\,[k-1]\cdots[1], & k \geq 1, \\ 1, & k = 0. \end{cases}$$