Anal. Theory Appl.
Vol. 27, No. 1 (2011), 51–58
DOI10.1007/s10496-011-0051-5

BOUNDEDNESS FOR THE COMMUTATOR OF FRACTIONAL INTEGRAL ON GENERALIZED MORREY SPACE IN NONHOMOGENEOUS SPACE

Guohua Liu and Lisheng Shu

(*Anhui Normal University, China)*

Received Mar. 12, 2010

Abstract. In this paper, we will establish the boundedness of the commutator generated by fractional integral operator and RBMO(μ) function on generalized Morrey space in the non-homogeneous space.

Key words: *fractional integral operator, commutator, generalized Morrey space, RBMO(μ)*

AMS (2010) subject classification: 42B20, 42B35

1 Introduction

Suppose μ is a non-negative Radon measure on $\mathbf{R}^d$ satisfying only the following growth condition: there exist constants $C>0$ and $n\in(0,d]$ such that for all $x\in\mathbf{R}^d$ and $r>0$

$$\mu(B(x,r))\leqslant Cr^n, \tag{1}$$

where

$$B(x,r)=\{y\in\mathbf{R}^d:|y-x|<r\}.$$

The Euclidean space $\mathbf{R}^d$ with a non-negative Radon measure only satisfying the growth condition is called a nonhomogeneous space.

In 2001, Tolsa developed a series of basic theory on nonhomogeneous space and introduced RBMO(μ) space. Recently, the properties of fractional integral commutator on Morrey space

Supported by the NSF of Education Committee of Anhui Province (KJ2011A138).

are studied in [2]. The purpose of this article is to establish the boundedness of the commutator generated by fractional integral operator and RBMO(μ) function on the generalized Morrey space in nonhomogeneous space. Before giving the main result, we introduce some necessary notations.

Let $(\mathbf{R}^d,\mu)$ be a nonhomogeneous space and Q a closed cube in $\mathbf{R}^d$ with sides parallel to the axes, we denote its sidelength by $l(Q)$. For $\alpha>0$, αQ stands for the cube with the same center as Q and having sidelength $\alpha l(Q)$. Given $\alpha>1,\beta>\alpha^n$, where n is the fixed number in growth condition , we say Q is a (α,β) doubling cube if $\mu(\alpha Q)\leqslant\beta\mu(Q)$. In the following, if α and β are not specified, by a doubling cube we mean a $(2,2^{d+1})$ doubling cube. Given two cubes $Q_1\subset Q_2$ in $\mathbf{R}^d$, we set

$$K_{Q_1,Q_2}=1+\sum_{k=1}^{N_{Q_1,Q_2}}\frac{\mu(2^kQ_1)}{l(2^kQ_1)^n},$$

where N_{Q_1,Q_2} is the first positive integer k such that $l(2^kQ_1)\geqslant l(Q_2)$.

Remark. In this paper, for $b\in L^1_{loc}(\mu)$, we denote the mean of b over the cube Q by m_Qb, that is ,

$$m_Qb=\frac{1}{\mu(Q)}\int_Q b(x)\,\mathrm{d}\mu(x).$$

Definition 1.[1] *Let $(\mathbf{R}^d,\mu)$ be a nonhomogeneous space, $b\in L^1_{loc}(\mu)$ and $\rho>1$ a fixed constant, we say that b is in* RBMO(μ), *if there exists a constant $C>0$ such that for any cube Q,*

$$\frac{1}{\mu(\rho Q)}\int_Q\left|b(x)-m_{\widetilde{Q}}b\right|\mathrm{d}\mu(x)\leqslant C, \tag{2}$$

and for any two doubling cubes $Q_1\subset Q_2$,

$$\left|m_{Q_1}b-m_{Q_2}b\right|\leqslant CK_{Q_1,Q_2}, \tag{3}$$

where $\widetilde{Q}$ is the smallest doubling cube in the family $\{2^kQ\}_{(k\in\mathbf{N})}$. The minimal constant C in (2) *and* (3) *is the* RBMO(μ) *norm of b, and it will be denoted by $\|b\|_*$.*

Remark. The definition of RBMO(μ) does not depend on the choice of ρ, see [1].

Definition 2.[2] *Let $(\mathbf{R}^d,\mu)$ be a nonhomogeneous space, n the fixed number in the growth condition and $0<s<n$, the fractional integral operator I_s on the nonhomogeneous space $(\mathbf{R}^d,\mu)$ is defined by*

$$I_sf(x)=\int_{\mathbf{R}^d}\frac{f(y)}{|x-y|^{n-s}}\,\mathrm{d}\mu(y). \tag{4}$$

Moreover, if $b\in$ RBMO(μ), *the commutator $[b,I_s]$ is defined by*

$$[b,I_s]f(x)=\int_{\mathbf{R}^d}\left[b(x)-b(y)\right]\frac{f(y)}{|x-y|^{n-s}}\,\mathrm{d}\mu(y). \tag{5}$$