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BOUNDS FOR COMMUTATORS OF MULTILINEAR FRACTIONAL INTEGRAL OPERATORS WITH HOMOGENEOUS KERNELS

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Abstract. We will show bounds for commutators of multilinear fractional integral operators with some homogeneous kernels.

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In 1999, C. E. Kenig and E. M. Stein[8] initiated the study of multilinear fractional integral operators defined as

$$I_\alpha(\vec{f})(x)=\int_{(\mathbf{R}^n)^m}\frac{1}{|(x-y_1,\cdots,x-y_m)|^{mn-\alpha}}\prod_{k=1}^m f_k(y_k)\mathrm{d}\vec{y}$$

(See [6] or [10] for more about fractional integral). Recently, K. Moen [11]m X. Chen and Q. Xue[3] developed the weighted theory for it, which was motivated by related research for multilinear singular integral in [7] and [9]. In their work the following of weights the for multilinear fractional integral was established.

Definition 1[11], [3]. Let $1\leqslant p_1,\cdots,p_m<\infty$, $\dfrac{1}{p}=\dfrac{1}{p_1}+\cdots+\dfrac{1}{p_m}$, and $q>0$. Suppose that $\vec{\omega}=(\omega_1,\cdots,\omega_m)$ and each ω_i $(i=1,\cdots,m)$ is a nonnegative function on $\mathbf{R}^n$. Then $\vec{\omega}\in A_{(\vec{p},q)}$

if it satisfies

$$\sup_Q \left(\frac{1}{|Q|}\int_Q \nu_{\vec{\omega}}{}^q\right)^{\frac{1}{q}} \prod_{i=1}^{m}\left(\frac{1}{|Q|}\int_Q \omega_i{}^{-p_i'}\right)^{\frac{1}{p_i'}} < \infty,$$

where $\nu_{\vec{\omega}} = \prod_{i=1}^{m} \omega_i$. If $p_i = 1$, $\left(\frac{1}{|Q|}\int_Q \omega_i^{-p_i'}\right)^{\frac{1}{p_i'}}$ is understood as $(\inf_Q \omega_i)^{-1}$.

Furthermore, a weighted norm inequality for multilinear fractional integral operators as below is proved.

Theorem A([11], [3])**.** *Let* $0 < \alpha < mn$, $1 < p_1, \cdots, p_m < \infty$, $\frac{1}{p} = \frac{1}{p_1} + \cdots + \frac{1}{p_m}$ *and* $\frac{1}{q} = \frac{1}{p} - \frac{\alpha}{n}$. *Then* $\vec{\omega} \in A_{(\vec{p},q)}$ *if and only if* I_α *can be extended to a bounded operator*

$$\left\|I_\alpha(\vec{f})\right\|_{L^q(\nu_{\vec{\omega}}{}^q)} \leqslant C\prod_{i=1}^{m}\left\|f_i\right\|_{L^{p_i}(\omega_i^{p_i})}. \tag{1}$$

In [3], besides the above, the authors proved another two results such as Theorem B and C, by the way of contemplating weighted norm inequalities for multilinear fractional integral with some homogeneous kernels and Coifman-Rochberg-Weiss commutators of multilinear fractional integral.

Theorem B[3]**.** *Let* $0 < \alpha < mn$, $1 \leqslant s' < p_1, \cdots, p_m < \infty$, $\frac{1}{p} = \frac{1}{p_1} + \cdots + \frac{1}{p_m}$ *and* $\frac{1}{q} = \frac{1}{p} - \frac{\alpha}{n}$. *Denote* $\vec{\omega}^{s'} = (\omega_1^{s'}, \cdots, \omega_m^{s'})$ *and* $\frac{\vec{p}}{s'} = (\frac{p_1}{s'}, \cdots, \frac{p_m}{s'})$. *Assume* $\vec{\omega}^{s'} \in A_{(\frac{\vec{p}}{s'}, \frac{q}{s'})} \bigcap A_{(\frac{\vec{p}}{s'}, \frac{q_\varepsilon}{s'})} \bigcap A_{(\frac{\vec{p}}{s'}, \frac{q_{-\varepsilon}}{s'})}$, *where* $\frac{1}{q_\varepsilon} = \frac{1}{p} - \frac{\alpha+\varepsilon}{n}$ *and* $\frac{1}{q_{-\varepsilon}} = \frac{1}{p} - \frac{\alpha-\varepsilon}{n}$. *Then, there exists a constant* $C > 0$ *independent of* $\vec{f}$ *such that*

$$\left\|I_{\Omega,\alpha}(\vec{f})\right\|_{L^q(\nu_{\vec{\omega}}{}^q)} \leqslant C\prod_{i=1}^{m}\left\|f_i\right\|_{L^{p_i}(\omega_i^{p_i})}, \tag{2}$$

where

$$I_{\Omega,\alpha}\vec{f}(x) = \int_{(\mathbf{R}^n)^m} \frac{\prod_{i=1}^{m}\Omega_i(x-y_i)f_i(y_i)}{|(x-y_1,\cdots,x-y_m)|^{mn-\alpha}}\,\mathrm{d}\vec{y}$$

and each $\Omega_i(x) \in L^s(\mathbf{S}^{n-1})$ $(i = 1, \cdots, m)$ *for some* $s > 1$ *is a homogeneous function with degree zero on* $\mathbf{R}^n$, *i.e. for any* $\lambda > 0$ *and* $x \in \mathbf{R}^n$, $\Omega_i(\lambda x) = \Omega_i(x)$.

Theorem C[3]**.** *Let* $0 < \alpha < mn$, $1 < p_1, \cdots, p_m < \infty$, $\frac{1}{p} = \frac{1}{p_1} + \cdots + \frac{1}{p_m}$ *and* $\frac{1}{q} = \frac{1}{p} - \frac{\alpha}{n}$. *For* $r > 1$ *with* $0 < r\alpha < mn$, *if* $\vec{\omega}^r \in A_{(\frac{\vec{p}}{r}, \frac{q}{r})}$ *and* $\nu_{\vec{\omega}}{}^q \in A_\infty$, *then there exists a constant* $C > 0$ *independent of* $\vec{b}$ *and* $\vec{f}$ *such that*

$$\left\|I_{\vec{b},\alpha}(\vec{f})\right\|_{L^q(\nu_{\vec{\omega}}{}^q)} \leqslant C\sup_i \|b_i\|_{\mathrm{BMO}}\prod_{i=1}^{m}\left\|f_i\right\|_{L^{p_i}(\omega_i^{p_i})}, \tag{3}$$