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APPROXIMATION AND SHAPE-PRESERVING PROPERTIES OF Q-STANCU OPERATOR

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Abstract. We introduce the definition of q-Stancu operator and investigate its approximation and shape-preserving property. With the help of the sign changes of $f(x)$ and $L_n = f(f,q;x)$ the shape-preserving property of q-Stancu operator is obtained.

Key words: *q-Stancu operator, shape-preserving property, sign change*

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1 Introduction

Suppose $q > 0$. For $k = 0, 1, 2, \cdots$, the q-integer $[k]$ and q-factorial $[k]!$ are defined as

$$[k] = \begin{cases} \frac{1-q^k}{1-q}, q \neq 1, \\ k, \quad q = 1; \end{cases}$$

$$[k]! = \begin{cases} [k][k-1]\dots[1], k \geq 1, \\ 1, \qquad k = 0. \end{cases}$$

For integers $n, k, n \geq k \geq 0$, q-binomial coeffcients are defined naturally as

$$\begin{bmatrix} n \\ k \end{bmatrix} = \frac{[n]!}{[k]![n-k]!}$$

We present the definition of q-Stancu operator as follows.

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Definition 1. *Suppose s is an integer and* $0 \leq s < \text{-}fracn2, q > 0, n > 0$. *For* $f \in C[0,1]$, *the operator*

$$L_n(f,q;x) = \sum_{k=0}^{n} f\left(\frac{[k]}{[n]}\right) b_{n,k,s}(q;x), \tag{1.1}$$

is called q-Stancu operator, where

$$b_{n,k,s}(q;x) = \begin{cases} (1-q^{n-k-s}x)p_{n-s,k}(q;x), & 0 \leq k < s, \\ (1-q^{n-k-s}x)p_{n-s,k}(q;x) + q^{n-k}xp_{n-s,k-s}(q;x), & s < k \leq n-s, \\ q^{n-k}xp_{n-s,k-s}(q;x), & n-s < k \leq n, \end{cases}$$

$p_{n-s,k}(q;x), p_{n-s,k-s}(q;x)$ are the basis functions of q-Bernstein operator,

$$p_{n,k}(q;x) = \begin{bmatrix} n \\ k \end{bmatrix} x^k \prod_{l=0}^{n-k-1} (1-q^l x)$$

It is not difficult to notice that on one hand for $s = 0$ or $s = 1$, q-Stancu operator is just the q-Bernstein operator which was introduced first by G.M. Phillips in 1997, on the other hand for $q = 1$, q-Stancu operator recoveres the Stancu operator. The q-Berstein operator possesses many remarkable properties which have made it an intensive area, seen [1-8]. While the study of Stancu operator is also a focus of many authors since 1981, after D.D. Stancu has defined this operator, see [9-12]. Both q-Bernstein operator and Stancu operator are some generalizations of the classical Bernstein operator which are specific cases of q-Bernstein operator when $q = 1$ or Stancu operator when $s = 0, s = 1$.

It is worth mentioning that the q-Stancu operator we defined here difier essentially from that in [13]. The q-Stancu operator in [13] just generalizes the control points of the Stancu operator based on the q-integers leaving alone the basis functions.While in our definition of q-Stancu operator both the control points and the basis functions are the q-analogue of those in Stancu operators. As a result, it is not a strange thing that these two q-Stancu operators behave quite difierent property, especially in the approximation problem.

By means of direct computations, we can get the following representation of q-Stancu operator:

$$L_n(f,q;x) = \sum_{k=0}^{n-s} \left\{ (1-q^{n-k-s}x) f\left(\frac{[k]}{[n]}\right) + q^{n-k-s}xf\left(\frac{[k+s]}{[n]}\right) \right\} p_{n-s,k}(q;x). \tag{1.2}$$

To process our study we need to give some essential properties of q-Stancu operator.

Proposition 1. *q-Stancu operator is a positive linear operator for* $0 < q < 1$, *while not for* $q > 1$.